Flipping Physics Lecture Notes:
Mechanics Free Response Question #2 Solutions
AP Physics C 1998 Released Exam from the College Board

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Circular orbit around Earth. 2 small spheres of mass *m*, centers connected by a rigid rod of length l with negligible mass. Small lump of clay of same mass *m* at speed v_o. Answers with: m, v_o, l and constants.

(a i) $v_i = 0$. Clay strikes at center of mass of the rod assembly, therefore the rod assembly will not rotate, therefore there will not be any rotational Kinetic Energy, only translational. During this collision all forces will be internal, therefore the net force will be zero, therefore momentum will be conserved.

$$\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow \vec{p}_{rod\,system\,initial} + \vec{p}_{clay\,initial} = \vec{p}_{rod\,system\,final} + \vec{p}_{clay\,final} \Rightarrow m_r\vec{v}_{ri} + m_c\vec{v}_{ci} = m_r\vec{v}_{rf} + m_c v_{cf}$$

$$\Rightarrow m_r\vec{v}_{ri} + m_c\vec{v}_{ci} = m_r\vec{v}_f + m_c\vec{v}_f = \left(m_r + m_c\right)\vec{v}_v \Rightarrow \left(m+m\right)\left(0\right) + \left(m\right)\left(v_o\right) = \left(\left(m+m\right)+m\right)v_f \Rightarrow v_f = \frac{mv_o}{3m} = \frac{v_o}{3}$$

Note: $\vec{v}_{rf} = \vec{v}_{cf} = \vec{v}_f$ because the clay is stuck to the rod system and therefore all the velocities will be the same.

$$KE_f = \frac{1}{2}m_{total}v_f^2 = \frac{1}{2}\left(3m\right)\left(\frac{v_o}{3}\right)^2 = \boxed{\frac{1}{6}mv_o^2}$$

(a ii) $\Delta KE = KE_f - KE_i = \frac{1}{6}mv_o^2 - \frac{1}{2}m_c v_{ci}^2 = \frac{1}{6}mv_o^2 - \frac{1}{2}mv_o^2 = \left(\frac{1}{6} - \frac{3}{6}\right)mv_o^2 = \boxed{-\frac{1}{3}mv_o^2}$

(b i) "Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres." This means that you can assume that the mass on the right and the clay lump have the same center of mass. Let's use the equation for the center of mass of a system of particles. Because the question asks the distance from the left mass, let's set the x zero position at the left mass.

$$x_{cm} = \frac{m_{left}x_{left} + m_{right}x_{right} + m_{clay}x_{clay}}{m_{left} + m_{right} + m_{clay}} = \frac{\left(m\right)\left(0\right) + \left(m\right)\left(l\right) + \left(m\right)\left(l\right)}{m+m+m} = \frac{2ml}{3m} = \boxed{\frac{2l}{3}}$$

(b ii) The initial linear momentum is up the page and all the forces are internal to the system, therefore the net force on the system is zero, therefore linear momentum is conserved, therefore the final momentum direction will be unchanged and be **up the page.**

(b iii) $\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow \vec{p}_{ri} + \vec{p}_{ci} = \vec{p}_{rf} + \vec{p}_{cf} \Rightarrow m_r\vec{v}_{ri} + m_c\vec{v}_{ci} = m_r\vec{v}_{rf} + m_c\vec{v}_{cf} = m_r\vec{v}_f + m_c\vec{v}_f = \left(m_r + m_c\right)\vec{v}_f$

$$\Rightarrow \left(m+m\right)\left(0\right) + \left(m\right)\left(v_o\right) = \left(\left(m+m\right)+m\right)v_f \Rightarrow v_f = \frac{mv_o}{3m} = \boxed{\frac{v_o}{3}}$$

(b iv) The net torque about the center of mass is zero, therefore angular momentum is conserved.

$$\left(\sum \tau_{center\,of\,mass} = 0 = \frac{d\vec{L}}{dt}\right)$$

$\sum \vec{L}_i = \sum \vec{L}_f \Rightarrow \vec{L}_{ri} + \vec{L}_{ci} = \vec{L}_{rf} + \vec{L}_{cf}$: The initial velocity of the rod assembly is zero so $\vec{L}_{ri} = 0$

Initially the clay lump is a particle so $\vec{L}_{ci} = \vec{r}_{ci} \times \vec{p}_{ci} = \vec{r}_{ci} m_c \vec{v}_{ci} \sin\theta_i = (\vec{r}_{ci} \sin\theta_i)(m_c \vec{v}_{ci}) = \left(\frac{l}{3}\right)(mv_o) = \frac{lmv_o}{3}$

Finally the clay lump is a part of the rod assembly, which is a rotation rigid object, therefore:

$\vec{L}_{rf} + \vec{L}_{cf} = \vec{L}_{total\,f} = I_{tf}\omega_f$

To find the total final moment of inertia, we need to use the equation for the moment of inertia for systems of particles:

$$I_{tf} = I_L + I_R + I_C = m_L r_L^2 + m_R r_R^2 + m_C r_C^2 = (m)\left(\frac{2l}{3}\right)^2 + (m)\left(\frac{l}{3}\right)^2 + (m)\left(\frac{l}{3}\right)^2$$

$$\Rightarrow I_{tf} = (m)\left(\frac{4l^2}{9}\right) + (m)\left(\frac{l^2}{9}\right) + (m)\left(\frac{l^2}{9}\right) = ml^2\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = \frac{2ml^2}{3}$$

$$\& \ \vec{L}_{total\,f} = I_{tf}\omega_f = \left(\frac{2ml^2}{3}\right)\omega_f$$

Returning to conservation of angular momentum:

$$\vec{L}_{ri} + \vec{L}_{ci} = \vec{L}_{total\,f} \Rightarrow 0 + \frac{lmv_o}{3} = \left(\frac{2ml^2}{3}\right)\omega_f \Rightarrow \omega_f = \boxed{\frac{v_o}{2l}}$$

(b v) This is similar to part (a ii), however, now there is a final rotational Kinetic Energy component.

$$\Delta KE = (KE_{tf} + KE_{rf}) - KE_{ti} = \frac{1}{2}m_{total}v_f^2 + \frac{1}{2}I_{tf}\omega_f^2 - \frac{1}{2}m_c v_{ci}^2 = \frac{1}{2}((m+m)+m)\left(\frac{v_o}{3}\right)^2 + \frac{1}{2}\left(\frac{2ml^2}{3}\right)\left(\frac{v_o}{2l}\right)^2 - \frac{1}{2}mv_o^2$$

$$\Rightarrow \frac{1}{2}(3m)\left(\frac{v_o^2}{9}\right) + \left(\frac{2ml^2v_o^2}{24l^2}\right) - \frac{1}{2}mv_o^2 = mv_o^2\left(\frac{1}{6} + \frac{1}{12} - \frac{1}{2}\right) = mv_o^2\left(\frac{2}{12} + \frac{1}{12} - \frac{6}{12}\right) = \boxed{-\frac{mv_o^2}{4}}$$

Notes about the published solutions:

(b i) This question is completely independent from part (a). This is not unusual. In other words, if you don't understand part of a problem, keep going, you may find a part that you do.

(b ii) No explanation necessary. Therefore, if you have no idea, please guess!

(b iv) The individual pieces of the whole conservation of angular momentum equation are worth points, not just the whole equation.