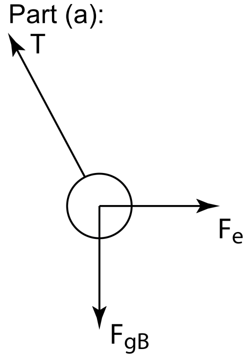
Flipping Physics Lecture Notes:
Electricity and Magnetism Free Response Question #1 Solutions
AP Physics C 1998 Released Exam from the College Board

$q_A = 120 \times 10^{-6}$ C, $m_B = 0.025$ kg, $r_B = 0.05$ m, θ = 20°, d = 1.5 m (distance between centers of mass).

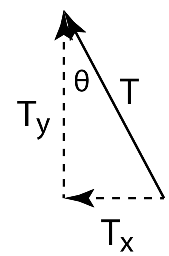
Part (a):



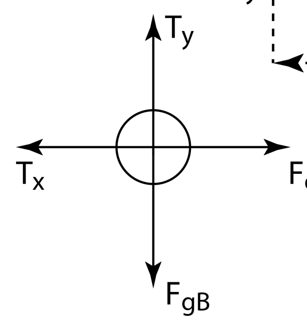
In the initial Free Body Diagram for Charge B_1, the Tension force act up in the direction of the string, the Force of Gravity acts down and the Electric Force (or Coulomb Force) is to the right (it must be to the right because it is pushing Charge B_1 to the right).

In order to sum the forces, we need to break the Tension into its components. $\sin\theta = \frac{O}{H} = \frac{T_x}{T} \Rightarrow T_x = T\sin\theta$ and

$$\cos\theta = \frac{A}{H} = \frac{T_y}{T} \Rightarrow T_y = T\cos\theta$$



Now we can re-draw the Free Body Diagram and sum the forces in the x and y directions.



$$\sum F_y = T_y - F_{gB} = m_B a_{By} = m_B(0) = 0 \Rightarrow T_y = F_{gB}$$

$$\Rightarrow T\cos\theta = m_B g \Rightarrow T = \frac{m_B g}{\cos\theta}$$

$$\sum F_x = F_e - T_x = m_B a_{Bx} = m_B(0) = 0 \Rightarrow T_x = F_e \Rightarrow T\sin\theta = \frac{kq_1q_2}{r^2}$$

$$\Rightarrow T\sin\theta = \frac{kq_Aq_B}{d^2} \Rightarrow \left(\frac{m_B g}{\cos\theta}\right)\sin\theta = m_B g\tan\theta = \frac{kq_Aq_B}{d^2} \Rightarrow q_B = \frac{m_B g d^2 \tan\theta}{kq_A}$$

$$\Rightarrow q_B = \frac{(0.025)(9.81)(1.5)^2\tan(20)}{(8.99\times10^9)(120\times10^{-6})} = 1.861729\times10^{-7}C \approx \boxed{1.86\times10^{-7}C}$$

(sorry, however, I refuse to round to 1 significant digit.)

Part (b): We have replaced Charge B with a conducting sphere with the same mass, radius and charge. Again the two spheres have found equilibrium and their *centers of mass* are d = 1.5 m apart. However, because Charge B is now a conductor, the charges can move and therefore Charge B will become slightly polarized. Because negative charges on Charge B will be closer to Sphere A than the positive charges on Charge B, the electric force, $F_e = \frac{kq_1q_2}{r^2}$, will be slightly reduced. The Tension in the x direction will still equal F_e and therefore the Tension, T, will be the smaller (because F_e was reduced). We know that $T = \frac{m_B g}{\cos\theta}$ and because m_B and g haven't changed, then $\boxed{\text{θ must also be smaller than 20°.}}$

Part (c): We now have a very long, horizontal, nonconducting tube with a radius, R = 0.20 m and a uniform linear charge density, $\lambda = +0.10\frac{\mu C}{m} = 1.0\times10^{-7}\frac{C}{m}$. We need to find the expression for the Electric field as a function of radius and they give us the answer, $E(r) = \frac{1800}{r}\frac{N}{C}$. In order to use Gauss' Law, $\Phi_E = \oint E\cdot dA = \frac{q_{in}}{\varepsilon_o}$, we start by drawing a Gaussian Surface. We need to draw it such that the Electric Field is constant on the Gaussian Surface and the angle between the Electric Field and the Gaussian Surface is either 0 or 90°. So, our Gaussian Surface needs to be a cylinder with radius r larger than R and a length l which is much smaller than the length of the tube.

$$\Phi_E = \oint_{total} E\cdot dA = \frac{q_{in}}{\varepsilon_o} \Rightarrow \oint_{side} E\cdot dA + \oint_{ends} E\cdot dA = \frac{q_{in}}{\varepsilon_o} \Rightarrow E\sin(0°)\oint_{side} dA + \oint_{ends} E\cos(90°)dA = \frac{q_{in}}{\varepsilon_o}$$

$$\Rightarrow E(A_{side}) = E\left[(2\pi r)(l)\right] = \frac{q_{in}}{\varepsilon_o} \text{ \& } \lambda = \frac{Q}{L} = \frac{q_{in}}{l} \Rightarrow q_{in} = \lambda l \Rightarrow E\left[(2\pi r)(l)\right] = \frac{\lambda l}{\varepsilon_o} \Rightarrow E = \frac{\lambda}{2\pi r\varepsilon_o}$$

$$\Rightarrow E = \frac{1.0\times10^{-7}}{(2\pi)(8.85\times10^{-12})r} = \frac{1798.36}{r} \approx \boxed{\frac{1800}{r}\frac{N}{C}}$$

Part (d): The "small" sphere A (i.e. point charge) with q = 120 x 10^{-6} C is r = 1.5 m from the center of the tube. F_e = ?

$$E = \frac{F_e}{q} \Rightarrow F_e = qE = q\left(\frac{1798.36}{r}\right) = (120\times10^{-6})\left(\frac{1798.36}{1.5}\right) = 0.143869 \approx \boxed{0.14N}$$

Part (e): Now we need to find the work done on sphere A to move it from r = 1.5 m to r = 0.30 m. Sphere A is always outside the tube (r > R), therefore our equation for the Electric Field is still valid.

$$W = \Delta U_{electric} = q\Delta V \text{ \& } E = -\frac{dV}{dr} \Rightarrow dV = -E\,dr \Rightarrow \int_{V_i}^{V_f} dV = -\int_{r_i}^{r_f} E\,dr \Rightarrow V_f - V_i = \Delta V = -\int_{1.5}^{0.3}\frac{1798.36}{r}dr$$

$$\Rightarrow \Delta V = -1798.36\int_{1.5}^{0.3}\frac{1}{r}dr = -1798.36\left[\ln r\right]_{1.5}^{0.3} = -1798.36\left[\ln(0.3) - \ln(1.5)\right] = 2894.35V$$

$$W = q\Delta V = (120\times10^{-6})(2894.35) = 0.347322 \approx \boxed{0.35J}$$