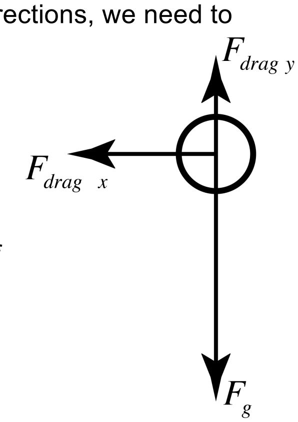
Flipping Physics Lecture Notes:
A Brief Look at the Force of Drag using
Numerical Modeling (or The Euler Method)

We did this example problem in the last lesson. *Example Problem:* While in a car moving at 10.0 miles per hour, mr.p drops a ball from a height of 0.70 m above the top of a bucket. How far in front of the bucket should he drop the ball such that the ball will land in the bucket?

Using the assumption that their was no air resistance, we used the concept of projectile motion to go from our initial known values to determine the displacement in the x-direction to be 1.68843 meters. In the video I stated that air resistance decreased the displacement in the x-direction by "less than 1 cm."

Let's talk about air resistance. When we add air resistance the object is no longer in projectile motion. In other words, the acceleration in the y-direction is *not* a constant -9.81 m/s^2 and the velocity in the x-direction is *not* constant. In order to figure out the acceleration of the ball in both the x and y directions, we need to draw a free body diagram of the forces acting on the ball while it is in the *air*.



The ball is moving to the right, so there will be a force of drag opposite that motion to the left. The ball is also moving down, so there will be a force of drag opposite that motion and down. These are actually the components of the net force of drag, however, it is easier to look at the drag in this case in terms of its components.

A typical equation for the force of drag is $F_{drag} = \frac{1}{2}\rho v^2 DA$, where ρ is *the density of the medium* through which the object is moving, v is the velocity of the object, D is called the Drag Coefficient and A is the cross sectional area of the object normal to the direction of travel. The density of air is not constant and is dependent on temperature and pressure, so, 1.275 kg/m^3 is a good approximation for today.[1] The cross section of a sphere in any direction is a circle. The ball that I used was a lacrosse ball for which there are clear specifications[2] from which you can determine that the radius of the ball is approximately 0.031835 m and the mass of the ball is approximately 0.14529 kg.

The drag coefficient of an object is defined by NASA as "a number which aerodynamicists use to model all of the complex dependencies of drag on shape, inclination, and some flow conditions."[3] Basically, it is an experimentally determined number that helps determine the drag on an object and changes depending on the shape of the object, the type of fluid flow around the object and the speed of the object. The drag coefficient of a baseball, according to NASA, is approximately 0.3, however, this isn't a baseball, so they alsp approximate a smooth sphere (like a lacrosse ball) to have a drag coefficient of around 0.5.[4] We will use 0.5 for the drag coefficient of our lacrosse ball.

A very important thing to notice about the drag force is that it is proportional to the square of the velocity. This means that as the velocity changes, the drag force changes, which changes the acceleration, which changes the velocity, which changes the drag force, which changes the acceleration, which changes the velocity … you get my point. This is certainly *not* uniformly accelerated motion, UAM, and the way we have to deal with it is by using Numerical Modeling other wise known as the Euler Method.

Leonhard Euler (1707-1783) devised this method of solving problems among other things, including finding Euler's number or the e number, 2.71828. Numerical Modeling or Euler's Method uses the idea that we can approximate the acceleration to be constant for a very short time interval and then we can use the UAM equations for that short time interval and come up with a good approximation of the motion. The shorter the time interval, the better the approximation.

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[1] http://chemistry.about.com/od/gases/f/What-Is-The-Density-Of-Air-At-Stp.htm
[2] http://www.uslacrosse.org/portals/1/documents/pdf/about-the-sport/nocsae-ball-standards.pdf
[3] http://www.grc.nasa.gov/WWW/k-12/airplane/shaped.html
[4] https://www.grc.nasa.gov/www/k-12/airplane/balldrag.html

Let's sum the forces in the x-direction and assume $v_{ix} = 10.0\frac{mi}{hr} \times \frac{1hr}{3600\sec} \times \frac{1609m}{1mi} = 4.469\overline{4}\frac{m}{s}$

$$\sum F_x = -F_{drag\,x} = ma_x \Rightarrow -\frac{1}{2}\rho_{air}v_{ix}^2DA = ma_x \Rightarrow a_x = -\frac{\rho_{air}v_{ix}^2D\left(\pi r^2\right)}{2m}$$

$$\Rightarrow a_x = -\frac{(1.275)\left(4.469\overline{4}\right)^2(0.5)\left(\pi(0.031835)^2\right)}{2(0.14529)} = -0.13953\frac{m}{s^2}$$

Now that we know the acceleration in the x-direction, we can approximate that acceleration to be constant for 1/100th of a second and determine v_{fx} using a UAM equation.

$$v_{fx} = v_{ix} + a_x\Delta t = \left(4.469\overline{4}\right) + (-0.13953)(0.01) = 4.46805\frac{m}{s}$$

To find the position in the x-direction after 1/100th of a second, we again use a UAM equation.

$$\Delta x = x_f - x_i = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2 \Rightarrow x_f = x_i + v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2 = (0) + \left(4.469\overline{4}\right)(0.01) + \frac{1}{2}(-0.13953)(0.01)^2$$

$\Rightarrow x_f = 0.04469m$ And these become the initial velocity and initial position for the next 1/100th of a second and we can determine the acceleration, then the final velocity, then the final position and this is why we have spreadsheets. We can Harness the Power of Excel!!

Let's look at the y-direction.

$$\sum F_y = F_{drag\,y} - F_g = \frac{1}{2}\rho_{air}v_{iy}^2DA - mg = ma_y \Rightarrow \frac{1}{2}\rho(0)^2DA - mg = ma_y \Rightarrow -mg = ma_y \Rightarrow a_y = -g = -9.81\frac{m}{s^2}$$

So at the very, very beginning, when the initial velocity in the y-direction equals zero, then the acceleration in the y-direction is -9.81 m/s^2, after that, the acceleration will begin to move toward zero.

To find the final velocity in the y-direction after 1/100th of a second, we again use a UAM equation.

$$v_{fy} = v_{iy} + a_y\Delta t = (0) + (-9.81)(0.01) = -0.0981\frac{m}{s}$$

To find the position in the x-direction after 1/100th of a second, we again use a UAM equation.

$$\Delta y = y_f - y_i = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow y_f = y_i + v_{iy}\Delta t + \frac{1}{2}a_x\Delta t^2 = (0) + (0)(0.01) + \frac{1}{2}(-9.81)(0.01)^2 = -4.905\times10^{-4}m$$

Now we move on to the next 1/100th of a second:

$$\sum F_y = F_{drag\,y} - F_g = \frac{1}{2}\rho v_{iy}^2DA - mg = ma_y \Rightarrow a_y = \frac{\rho v_{iy}^2D\pi r^2}{2m} - g$$

$$\Rightarrow a_y = \frac{(1.275)(-0.0981)^2(0.5)\left(\pi(0.031835)^2\right)}{2(0.14529)} - 9.81 = -9.80993\frac{m}{s^2}$$

And again we Harness the Power of Excel!! And you can see that, when we include air resistance, the ball will move 1.67984 and not 1.68843 meters in the x-direction. Which is a difference of:

$1.68843 - 1.67984 = 0.00859m \times \frac{1000mm}{1m} = 8.59mm \approx \boxed{9mm}$, which is less than 1 cm.

The Excel file I refer to can be found @ www.flippingphysics.com/euler-method.html