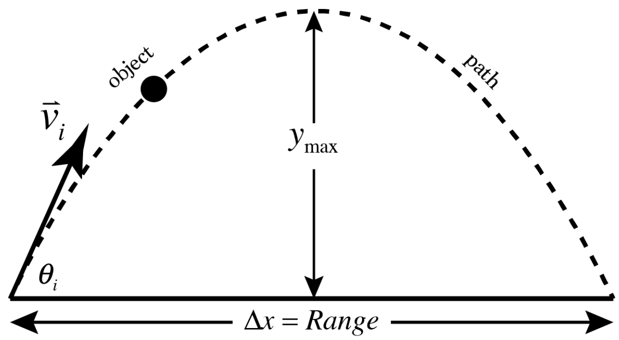
Flipping Physics Lecture Notes:
Understanding the Range Equation of Projectile Motion

The range of an object in projectile motion means something very specific. It is the displacement in the x direction of an object whose displacement in the y direction is zero.



Students often get confused by the statement "displacement in the y direction is zero" or Δy = 0. This does *not* mean that the object does not move up or down, it simply means that it ends at the same height it started as: $\Delta y = y_f - y_i = 0$

The Range Equation is $\boxed{R = \frac{v_i^2 \sin(2\theta_i)}{g}}$ & the variables in the range equation are:

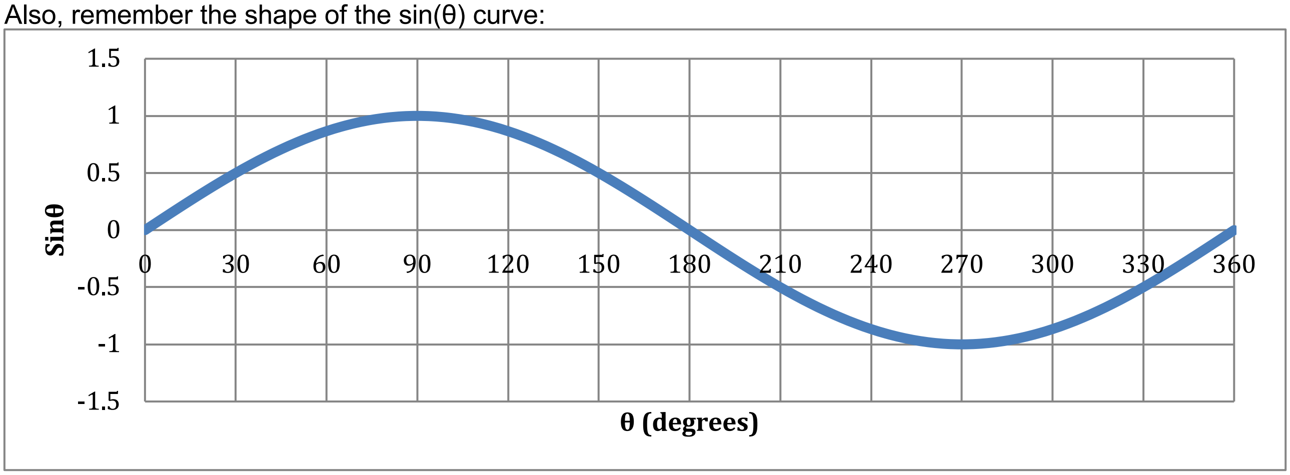
- $\Delta x = Range = R$ (in other words, "R", stands for Range. Needs to be in meters.)
- $v_i \Rightarrow \left\| v_i \right\|$ (the magnitude of the initial velocity. Needs to be in meters per second.)
- $\theta_i \Rightarrow$ (the initial angle or launch angle. Usually in degrees & has to match your calculator mode.)
- $g_{Earth} = +9.81\frac{m}{s^2}$ (remember g is a *positive* number)

We can determine the angle that will give the largest range with the same magnitude initial velocity by remembering that the maximum value for the sine of any angle is 1:

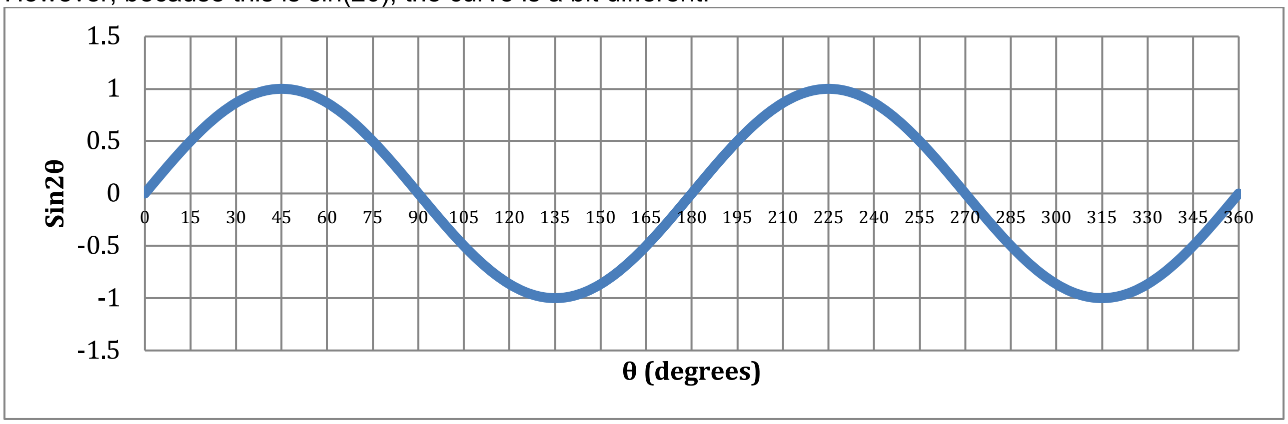
$$\text{Max Value} = 1 = \sin(2\theta_i) \Rightarrow 2\theta_i = \sin^{-1}(1) = 90° \Rightarrow \theta_i = \frac{90°}{2} = 45°$$

Therefore, the maximum range is when θ_i is 45°

Also, remember the shape of the sin(θ) curve:

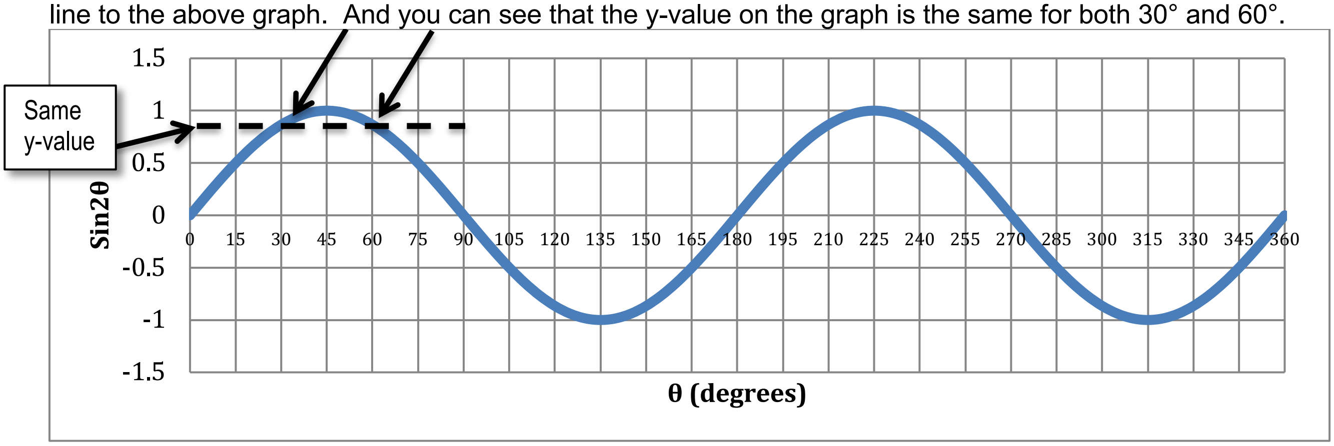


However, because this is sin(2θ), the curve is a bit different:



This means that $\sin(2\theta_i) = \sin(2(90 - \theta_i))$. Which means that there are two different angles that will have the same range. For example, $\theta_{1i} = 30°$ & $\theta_{2i} = (90 - \theta_{1i}) = (90 - 30) = 60°$, because both have the same value for $\sin(2\theta_i)$:

You can also see that $\sin(2\theta_i) = \sin(2(90 - \theta_i))$ or $\sin(2\theta_i) = \sin(2(90 - \theta_i))$ when I add a horizontal line to the above graph. And you can see that the y-value on the graph is the same for both 30° and 60°.



Just so you know, θ_{1i} & θ_{2i} are complementary angles because they add up to 90°. So two launch angles that are complementary will result in the same range.

Now, θ_{1i} & θ_{2i} will both result in the same range, however, $\theta_{2i} = 60°$ with go higher and be in the air longer than $\theta_{1i} = 30°$. Which can be seen in the figure below. (By "air" or course, I mean "the vacuum you can breathe".)