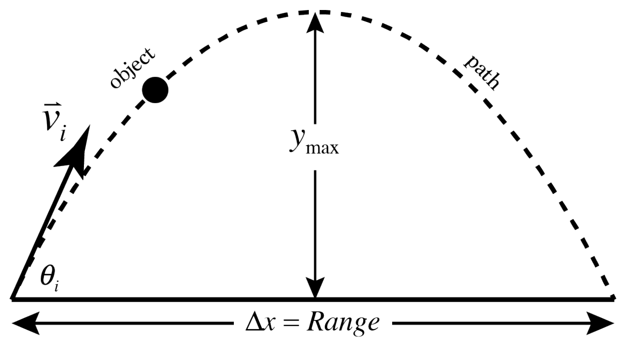
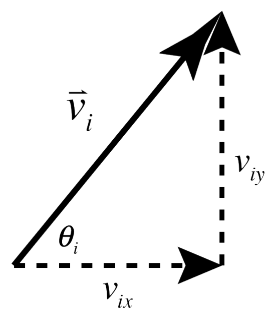
Flipping Physics Lecture Notes:
Deriving the Range Equation of Projectile Motion



The range of an object in projectile motion means something very specific. It is the displacement in the x direction of an object whose displacement in the y direction is zero. $\Delta x = Range = R$
(in other words, "R", stands for Range.)

The Range Equation or $R = \dfrac{v_i^2 \sin(2\theta_i)}{g}$ can be derived from the projectile motion equations. We start by breaking our initial velocity in to its components and then list everything we know in the x and y directions:



$$\sin\theta = \frac{O}{H} \Rightarrow \sin\theta_i = \frac{v_{iy}}{v_i} \Rightarrow v_{iy} = v_i \sin\theta_i \ \&$$

$$\cos\theta = \frac{A}{H} \Rightarrow \cos\theta_i = \frac{v_{ix}}{v_i} \Rightarrow v_{ix} = v_i \cos\theta_i = v_x$$

Remember that in the x-direction an object in projectile motion has a constant velocity, therefore $v_{ix} = v_x$.

x-direction: $v_{ix} = v_i \cos\theta_i = v_x$, $\Delta x = R = ?$

y-direction: $\Delta y = 0$ & $a_y = -g$ (remember $g_{Earth} = +9.81\frac{m}{s^2}$)

Let's start in the x-direction where there is a constant velocity and solve for the Range.

$$v_x = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = R = (\Delta t)v_x = (\Delta t)v_i \cos\theta_i$$

Now we need to solve for Δt in the y-direction and substitute Δt in to $R = (\Delta t)v_i \cos\theta$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2 = 0 \Rightarrow 0 = v_{iy} + \frac{1}{2}a_y \Delta t \Rightarrow v_{iy} = -\frac{1}{2}a_y \Delta t = -\frac{1}{2}(-g)\Delta t = \frac{1}{2}g\Delta t$$

$$\Rightarrow 2v_{iy} = g\Delta t \Rightarrow \Delta t = \frac{2v_{iy}}{g} = \frac{2v_i \sin\theta_i}{g}$$

And now we can substitute back in.

$$R = (\Delta t)v_i \cos\theta_i = \left(\frac{2v_i \sin\theta_i}{g}\right)v_i \cos\theta_i = \frac{v_i^2(2\sin\theta_i \cos\theta_i)}{g} \Rightarrow \boxed{R = \frac{v_i^2 \sin(2\theta_i)}{g}}$$

This uses the sine double angle formula from trig: $2\sin\theta_i \cos\theta_i = \sin(2\theta_i)$

FYI: It is generally *not* assumed that students in an algebra based physics class will know or remember various trig functions like this.