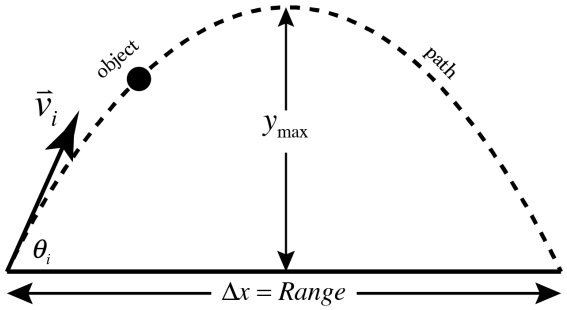
Flipping Physics Lecture Notes:
A Range Equation Problem with Two Parts

Example Problem: Mr.p throws a ball toward a bucket that is 581 cm away from him horizontally. He throws the ball at an initial angle of 55° above the horizontal and the ball is 34 cm short of the bucket. If mr.p throws the ball with the same initial speed and the ball is always released at the same height as the top of the bucket, at what angle does he need to throw the ball so it will land in the bucket?



Attempt #1: The range of the first throw is 34 cm short of the bucket.

So $R_1 = 581 - 34 = 547cm \times \frac{1m}{100cm} = 5.47m;\ \theta_{1i} = 55°;\ g = +9.81\frac{m}{s^2};\ v_{1i} = ? = v_{2i} = v_i\ \&\ \theta_{2i} = ?$

Now we need to solve the Range equation for the magnitude of the initial velocity:

$$R_1 = \frac{v_i^2 \sin(2\theta_{1i})}{g} \Rightarrow gR_1 = v_i^2 \sin(2\theta_{1i}) \Rightarrow v_i^2 = \frac{gR_1}{\sin(2\theta_{1i})} \Rightarrow v_i = \sqrt{\frac{gR_1}{\sin(2\theta_{1i})}}$$

$$\Rightarrow v_i = \sqrt{\frac{(9.81)(5.47)}{\sin(2\cdot 55)}} = 7.55675\frac{m}{s}$$

For the 2nd attempt, we use the same initial velocity magnitude, we know the range for the 2nd attempt, $R_2 = 581cm \times \frac{1m}{100cm} = 5.81m$, and now we solve the Range Equation for the angle:

$$R_2 = \frac{v_i^2 \sin(2\theta_{2i})}{g} \Rightarrow gR_2 = v_i^2 \sin(2\theta_{2i}) \Rightarrow \sin(2\theta_{2i}) = \frac{gR_2}{v_i^2} \Rightarrow \sin^{-1}\left(\sin(2\theta_{2i})\right) = 2\theta_{2i} = \sin^{-1}\left(\frac{gR_2}{v_i^2}\right)$$

$$\Rightarrow 2\theta_{2i} = \sin^{-1}\left(\frac{(9.81)(5.81)}{(7.55675)^2}\right) = 86.46870° \Rightarrow \theta_{2i} = \frac{86.46870°}{2} = 43.23435° \approx \boxed{43°}$$

Please note: As shown in "Understanding the Range Equation of Projectile Motion", there are actually two angles that will result in the same range and they are complementary angles, therefore:

$$\theta_{2i} = 43.23435°\ or\ (90° - 43.23435°) = 43.23435°\ or\ 46.76565° \approx \boxed{43°\ or\ 47°}$$

However, many physics teachers will not require that you understand that there are two angles. Why not? Because knowing that there are two angles with the same range has more to do with your understanding trigonometry than physics.