Flipping Physics Lecture Notes:
The Classic Bullet Projectile Motion Experiment

Example Problem: One bullet is fired horizontally and simultaneously a second bullet is dropped from the same height. Neglecting air resistance and assuming the ground is level, which bullet hits the ground first?

Let's list the known Uniformly Accelerated Motion, UAM, *y-direction* variables for both bullets:

Dropped:	*Fired Horizontally:*	
$\Delta y = -h$	$\Delta y = -h$	The Same
$v_{iy} = 0$	$v_{iy} = 0$	The Same
$a_y = -g$	$a_y = -g$	The Same

If three of the UAM variables are the same, then so are the other two: Δt and v_{fy} are the same for both bullets. Mathematically, it looks like this:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow -h = (0)\Delta t + \frac{1}{2}(-g)\Delta t^2 \Rightarrow \Delta t^2 = \frac{2h}{g} \Rightarrow \Delta t = \sqrt{\frac{2h}{g}} \text{ (for both)}$$

$$v_{fy} = v_{iy} + a_y\Delta t = 0 + (-g)\left(\sqrt{\frac{2h}{g}}\right) = -\sqrt{\frac{2g^2h}{g}} = \sqrt{2gh} \text{ (for both)}$$

Note: Even though both bullets will strike the ground at the same time and have the same final velocity in the y-direction, the bullet that was fired horizontally will be moving *faster*. This is because it will also have a velocity in the x-direction, and the final velocity will be the resultant vector of both the x and y-direction final velocities. Because the hypotenuse of a right triangle is, by definition, longer than either of the two sides, the one fired horizontally will have a larger magnitude final velocity and, therefore, will be moving faster than the one that was dropped.

Dropped:	*Fired Horizontally:*