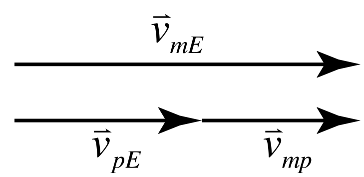
Flipping Physics Lecture Notes:
Introduction to Relative Motion using a Quadcopter Drone

Up to this point all velocities have been understood to be relative to the ground or the Earth. And, unless otherwise noted, that's the way it will continue. However, today we are going to look at velocities relative to objects that are not the Earth. Let's start with the velocities of two cars relative to the Earth:

$\vec{v}_{mE} = 24\frac{mi}{hr}E$ (Read: The velocity of the minivan with respect to the Earth is 24 miles per hour East)

$\vec{v}_{pE} = 13\frac{mi}{hr}E$ (Read: The velocity of the prius with respect to the Earth is 13 miles per hour East)

Now let's find the velocity of the minivan with respect to the prius: $\vec{v}_{mp} = ?$ In other words, while I am driving the prius and I look out the window to watch the minivan go by, at what velocity would I measure the minivan to be moving? Some of you may be able to immediately identify that the answer is 11 miles per hour East, however, in order to understand more complicated relative motion problems, let's walk through the math. The vectors look like this:



From the vector diagram you can see that $\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp}$ which we can solve for $\vec{v}_{mp}$:

$$\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp} \Rightarrow \vec{v}_{mE} - \vec{v}_{pE} = \vec{v}_{mp} \Rightarrow \vec{v}_{mp} = \vec{v}_{mE} - \vec{v}_{pE} = 24\frac{mi}{hr}E - 13\frac{mi}{hr}E = \boxed{11\frac{mi}{hr}E}$$

You should remember that taking the negative of a vector changes the direction by 180°. In terms of relative motion, it switches the order of the subscripts. In other words: $-\vec{v}_{pE} = \vec{v}_{Ep} = -13\frac{mi}{hr}E = 13\frac{mi}{hr}W$ Which is useful because it means we can substitute $\vec{v}_{Ep}$ into the equation: $\vec{v}_{mp} = \vec{v}_{mE} - \vec{v}_{pE} \Rightarrow \vec{v}_{mp} = \vec{v}_{mE} + \vec{v}_{Ep}$ and notice what happens to the subscripts on the right hand side of the equation; when you add two vectors together like this, the Earth drops out.

Read that last equation very carefully, $\vec{v}_{mp} = \vec{v}_{mE} + \vec{v}_{Ep}$: The velocity of the minivan with respect to the prius is the same as the velocity of the minivan with respect to the Earth plus the velocity of the Earth with respect to the prius. Understanding that the Earth drops out of the equation will make more complicated relative motion problems easier.

The same is actually true of the first equation: $\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp}$, The velocity of the prius drops out of the right hand side of the equation to give us the velocity of the minivan with respect to the Earth.

Now let's find the velocity of the prius with respect to the minivan, $\vec{v}_{pm} = ?$ In other words, while my wife drives the minivan and she looks out the window to watch the prius as she passes it, at what velocity would she measure the prius to be moving? The solution is actually rather simple: $\vec{v}_{pm} = -\vec{v}_{mp} = -11\frac{mi}{hr}E = \boxed{11\frac{mi}{hr}W}$