

Flipping Physics Lecture Notes: An introductory Relative Motion Problem with Vector Components

Example Problem: A toy car travels at 42 mm/s @ 18° E of N relative to a piece of paper that is moving at 71 mm/s W relative to the Earth. What is the velocity of the toy car relative to the Earth?

Givens:
$$\vec{v}_{cp} = 42 \frac{mm}{s} @ 18^{\circ} E of N$$
, $\vec{v}_{pE} = 71 \frac{mm}{s} W$, $\vec{v}_{cE} = ?$
 $\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE}$

(The velocity of the car with respect to the Earth is the same as the velocity of the car with respect to the paper plus the velocity of the paper with respect to the Earth; the paper drops out of the equation.)



We can't use Pythagorean theorem or SOH CAH TOA because we don't have a right triangle. We need to resolve or break \vec{v}_{cp} in to its components first.

$$\vec{v}_{cp_{x}} = \sin \theta = \frac{0}{H} = \frac{v_{cp_{x}}}{v_{cp}} \Rightarrow v_{cp_{x}} = v_{cp} \sin \theta = (42) \sin(18) = 12.979 \frac{mm}{s}$$

$$\vec{v}_{cp_{y}} = \vec{v}_{cp_{y}} \Rightarrow v_{cp_{y}} \Rightarrow v_{cp_{y}} = v_{cp} \cos \theta = (42) \cos(18) = 39.944 \frac{mm}{s}$$
Now we need to redraw the vector diagram.
And you can see that we now have a right triangle.

$$\vec{v}_{pE} = v_{cE} = (v_{pE} + v_{cp_{x}})^{2} + v_{cp_{y}}^{2} \Rightarrow v_{cE} = \sqrt{(v_{pE} + v_{cp_{x}})^{2} + v_{cp_{y}}^{2}}$$

$$\Rightarrow v_{cE} = \sqrt{(-71 + 12.979)^{2} + (39.944)^{2}} = 70.442 \approx 7.0 \times 10^{1} \frac{mm}{s}$$

$$a^{2} + b^{2} = c^{2} \Longrightarrow v_{cE}^{2} = \left(v_{pE} + v_{cp_{x}}\right)^{2} + v_{cp_{y}}^{2} \Longrightarrow v_{cE} = \sqrt{\left(v_{pE} + v_{cp_{x}}\right)^{2} + v_{cp_{y}}^{2}}$$
$$\Longrightarrow v_{cE} = \sqrt{\left(-71 + 12.979\right)^{2} + \left(39.944\right)^{2}} = 70.442 \approx 7.0 \times 10^{1} \frac{mm_{x}}{s}$$

That is only the magnitude, now we need the direction.

$$\cos\theta = \frac{A}{H} = \frac{v_{cp_y}}{v_{cE}} \Rightarrow \theta = \cos^{-1}\left(\frac{v_{cp_y}}{v_{cE}}\right) = \cos^{-1}\left(\frac{39.944}{70.442}\right) = 55.455 \approx 55^{\circ}$$
$$\Rightarrow \vec{v}_{cE} \approx \boxed{7.0 \times 10^1 \frac{mm}{s} @ 55^{\circ} W \text{ of } N}$$

 (\vec{v}_{cp_y})