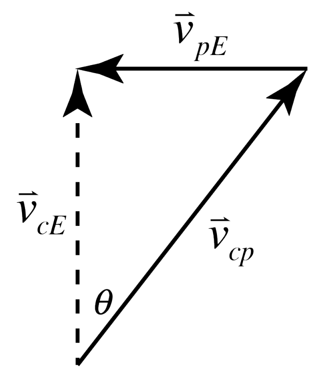
Flipping Physics Lecture Notes:
Relative Motion Problem: Solving for the angle of the moving object

Example Problem: A toy car is moving on a piece of paper. The toy car can travel at a speed of 47 mm/s relative to the paper. The paper has a velocity of 29 mm/s W relative to the Earth. (a) At what angle should the toy car travel relative to the paper such that the car will move due north relative to the Earth? (b) What is the speed of the car relative to the Earth?

Givens: $\vec{v}_{cp} = 47\frac{mm}{s}$ @ $\theta = ?$, $\vec{v}_{pE} = 29\frac{mm}{s}W$, $\vec{v}_{cE}$ is North.

Because the paper is traveling west and the velocity of the car with respect to the Earth is North, the car must be traveling in the northeasterly direction, we just need to figure out exactly what direction. The vector diagram looks like this:



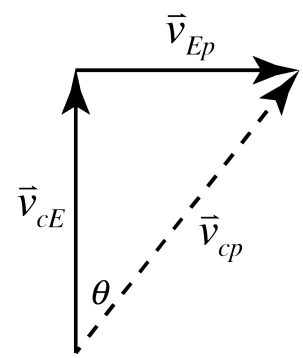
We get the same vector addition formula we have had now three times in a row: $\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE}$ However, now we are not solving for $\vec{v}_{cE}$, we are solving for the direction of $\vec{v}_{cp}$. Which means we need to rearrange the formula.

$$\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE} \Rightarrow \vec{v}_{cp} = \vec{v}_{cE} - \vec{v}_{pE}$$

And remember $\vec{v}_{Ep} = -\vec{v}_{pE}$ so: $\vec{v}_{cp} = \vec{v}_{cE} - \vec{v}_{pE} \Rightarrow \vec{v}_{cp} = \vec{v}_{cE} + \vec{v}_{Ep}$

(The velocity of the car with respect to the paper is the same as the velocity of the car with respect to the Earth plus the velocity of the Earth with respect to the paper. Remember, the Earth drops out of the equation when it is the middle subscript.)

Remembering that switching the order of the subscripts rotates the vector 180°, we can draw a new velocity vector diagram:



Now we have a right triangle, vector addition problem.

$$\sin\theta = \frac{O}{H} = \frac{v_{Ep}}{v_{cp}} \Rightarrow \theta = \sin^{-1}\left(\frac{v_{Ep}}{v_{cp}}\right) = \sin^{-1}\left(\frac{29}{47}\right) = 38.099 \approx \boxed{38^\circ\ E\,of\,N}$$

Part (b)

$$a^2 + b^2 = c^2 \Rightarrow v_{cE}^2 + v_{Ep}^2 = v_{cp}^2 \Rightarrow v_{cE}^2 = v_{cp}^2 - v_{Ep}^2 \Rightarrow v_{cE} = \sqrt{v_{cp}^2 - v_{Ep}^2} = \sqrt{47^2 - 29^2} = 36.986\frac{mm}{s} \approx \boxed{37\frac{mm}{s}}$$

You may have noticed that the length of the vectors here and in the video don't quite match. Sadly, the velocity of the paper with respect to the Earth was actually 39 mm/s (which is what I used in the video illustrations to match the demonstration), however, at some point between filming the demonstration and writing the script and lecture notes, it got changed to 29 mm/s. I didn't notice until I was *completely* done with the video. Because the physics learning is unaffected and unlimited resources and time are decidedly *not* at my disposal, I made the difficult decision not to refilm the entire video. I hope this in no way tarnishes your respect for me. ☺