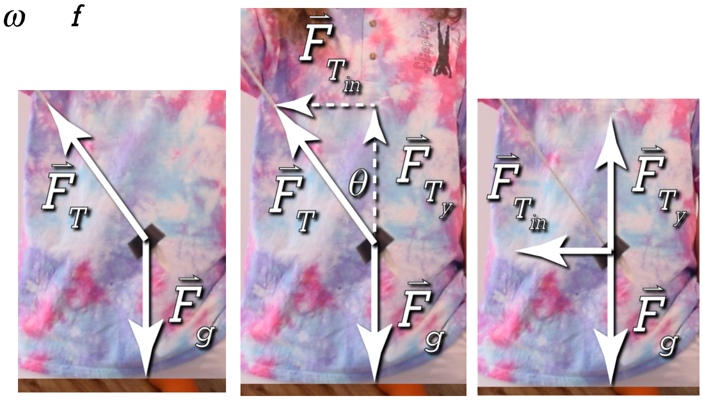
Flipping Physics Lecture Notes:
AP Physics 1 Review of *Rotational Kinematics*
https://www.flippingphysics.com/ap1-rotational-kinematics-review.html

- Angular Velocity: $\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}\left(\frac{rad}{s} \text{ or } \frac{rev}{min}\right)$
    - Remember for conversions: $1\ rev = 360° = 2\pi\ radians$
- Angular Acceleration: $\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}\left(\frac{rad}{s^2}\right)$
- Uniformly Angularly Accelerated Motion, UαM, is just like UAM, only it uses angular variables:
    - Equations are valid when $\vec{\alpha} = constant$

Uniformly Accelerated Motion, UAM	Uniformly Angularly Accelerated Motion, UαM
$v_x = v_{x0} + a_x t$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v_x{}^2 = v_{x0}{}^2 + 2a_x\left(x - x_0\right)$	$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\Delta\theta$
$\Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$	$\Delta\theta = \frac{1}{2}\left(\omega_f + \omega_i\right)\Delta t$

- Tangential velocity is the linear velocity of an object moving along a circular path. $\vec{v}_t = r\vec{\omega}$
    - The direction of tangential velocity is tangent to the circle and normal to the radius.
    - Tangential velocity is a linear velocity so it has the same dimensions as linear velocity: $\frac{m}{s}$
- Centripetal Force and Centripetal Acceleration:
    - Centripetal means “Center Seeking”
    - Centripetal force is the net force in the in direction or the “center seeking” force which causes the acceleration of the object in toward the center of the circle which is the centripetal or “center seeking” acceleration.
    - Centripetal Force, $\sum \vec{F}_{in} = m\vec{a}_c$:
        - Not a new force.
        - Never in a Free Body Diagram.
        - The direction “in” is positive and the direction “out” is negative.
    - Centripetal Acceleration, $a_c = \frac{v_t{}^2}{r} = r\omega^2$
- The Period, T, is the time for one full cycle or revolution.
    - Dimensions for period: seconds or seconds per cycle.
- The Frequency, f, is the number of cycles or revolutions per second.
    - Dimensions for frequency are cycles per second which are called Hertz, Hz: $f \Rightarrow \frac{cyc}{\text{sec}} = Hz$
    - Frequency and Period are inversely related: $T = \frac{1}{f}$

- We can use the equation for angular velocity to derive an equation on the equation sheet:
  - $\vec{\omega} = \dfrac{\Delta\vec{\theta}}{\Delta t} = \dfrac{2\pi\ rad}{T} \Rightarrow T = \dfrac{2\pi}{\omega} = \dfrac{1}{f}$
- The Conical Pendulum Example:



$$\sin\theta = \frac{O}{H} = \frac{\vec{F}_{T_{in}}}{\vec{F}_T} \Rightarrow \vec{F}_{T_{in}} = \vec{F}_T \sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{\vec{F}_{T_y}}{\vec{F}_T} \Rightarrow \vec{F}_{T_y} = \vec{F}_T \cos\theta$$

$$\sum F_y = F_{T_y} - F_g = ma_y = m(0)$$

$$\Rightarrow F_{T_y} = F_T \cos\theta = mg$$

$$\sum F_{in} = F_{T_{in}} = \vec{F}_T \sin\theta = ma_c = m\left(\frac{v_t^{\,2}}{r}\right)$$

$$v_t = r\omega \text{ or } v_t = \frac{\Delta x}{\Delta t} = \frac{C}{T} = \frac{2\pi r}{T}$$

We could even substitute further:

$$F_T \cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\vec{F}_T \sin\theta = m\left(\frac{v_t^{\,2}}{r}\right) \Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = m\left(\frac{\left(\frac{2\pi r}{T}\right)^2}{r}\right) \Rightarrow g\tan\theta = \frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 r}{T^2}$$

And solve for the radius in terms of the length of the string.

$$\sin\theta = \frac{O}{H} = \frac{r}{L} \Rightarrow r = L\sin\theta$$

$$g\tan\theta = \frac{4\pi^2 r}{T^2} \Rightarrow g\frac{\sin\theta}{\cos\theta} = \frac{4\pi^2 L\sin\theta}{T^2} \Rightarrow \frac{g}{\cos\theta} = \frac{4\pi^2 L}{T^2} \Rightarrow \boxed{T^2 = \frac{4\pi^2 L\cos\theta}{g}}$$

And we end with an expression for the period of the circular motion.