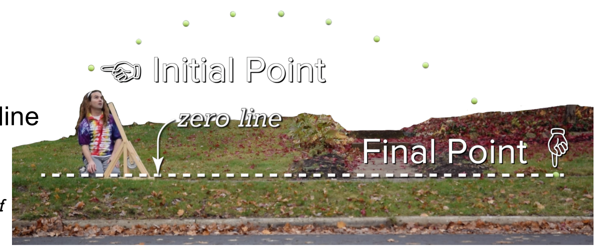
Flipping Physics Lecture Notes:

Introductory Conservation of Mechanical Energy Problem

Example: A tennis ball with a mass of 58 grams is launched from a trebuchet with an initial speed of 6.8 m/s and an initial height of 1.3 meters. Assuming level ground, what is the final speed of the ball right before it strikes the ground?

$$m = 58g;\ v_i = 6.8\frac{m}{s};\ h_i = 1.3m;\ v_f = ?$$



Identify initial and final points and set the horizontal zero line

$$ME_i = ME_f \Rightarrow KE_i + PE_{gi} + PE_{ei} = KE_f + PE_{gf} + PE_{ef}$$

$$\Rightarrow \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

$$\Rightarrow \frac{1}{2}mv_i^2 + mgh_i + \cancel{\frac{1}{2}kx_i^2} = \frac{1}{2}mv_f^2 + \cancel{mgh_f} + \cancel{\frac{1}{2}kx_f^2}$$

No Elastic Potential Energy at all because there is no spring and no Gravitational Potential Energy final because the vertical height final above the horizontal zero line is zero.

$$\Rightarrow \frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^2 \Rightarrow \frac{1}{2}v_i^2 + gh_i = \frac{1}{2}v_f^2$$ Everybody brought mass to the party!

$$\Rightarrow v_i^2 + 2gh_i = v_f^2 \Rightarrow v_f = \sqrt{v_i^2 + 2gh_i} = \sqrt{6.8^2 + (2)(9.81)(1.3)} = 8.4703 \approx \boxed{8.5\frac{m}{s}}$$

Note: This is the final *speed* of the ball, not the final velocity. Mechanical Energy is a scalar and therefore we can only solve for the magnitude of the final velocity.

Also note: We couldn't solve this problem using projectile motion because we did not have an initial angle for the projectile.

In case you were curious, I actually used projectile motion equations to determine the initial speed for this problem. I measured the vertical and horizontal displacements and, if you count frames, there are 57 frames from launch until landing, at 60 frames per second:

$$\Delta t_t = 57 frames \times \frac{1 sec}{60 frames} = \frac{57}{60}s;\ \Delta x_t = 5.64m;\ \Delta y_t = -1.34m$$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2 \Rightarrow -1.34 = v_{iy}\left(\frac{57}{60}\right) + \frac{1}{2}(-9.81)\left(\frac{57}{60}\right)^2 \Rightarrow v_{iy} = 3.24922\frac{m}{s}$$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{5.64}{57/60} = 5.93684\frac{m}{s}\ \&\ a^2 + b^2 = c^2 \Rightarrow v_i^2 = v_{iy}^2 + v_{ix}^2 \Rightarrow v_i = \sqrt{v_{iy}^2 + v_{ix}^2}$$

$$v_i = \sqrt{3.24922^2 + 5.93684^2} = 6.767832 \approx 6.8\frac{m}{s}$$ is the magnitude of the initial velocity.

Yes, we could use $\tan\theta = \frac{O}{H} = \frac{v_{iy}}{v_{ix}}$ to find the initial launch angle.