Flipping Physics Lecture Notes:

Conservation of Energy Problem with Friction, an Incline and a Spring

by Billy

Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μ_k between the block and the incline is 0.30, to what maximum height does the block slide?

Givens: $k = 14\frac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{max} = ?$

Convert knowns to base SI units:

$$m = 11g \times \frac{1kg}{1000g} = 0.011kg \ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

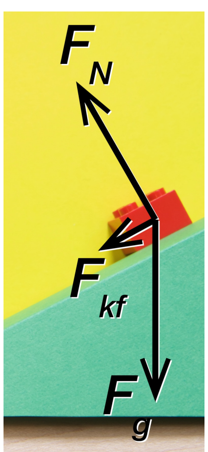
On the level surface, there is no work done by friction or the force applied; therefore we can use Conservation of Mechanical Energy. Set the initial point where the block is completely compressing the spring, the final point at the base of the incline and the zero line at the center of mass of the block while it is on the incline.

$$ME_i = ME_f \Rightarrow \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

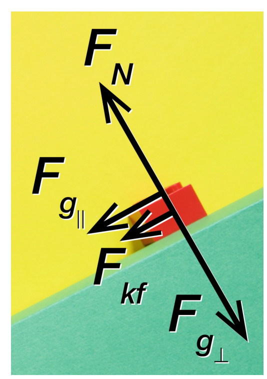
$$\Rightarrow \underbrace{\cancel{\frac{1}{2}mv_i^2}}_{v_i = 0} + \underbrace{\cancel{mgh_i}}_{h_i = 0} + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \underbrace{\cancel{mgh_f}}_{h_i = 0} + \underbrace{\cancel{\frac{1}{2}kx_f^2}}_{\text{not on spring}} \Rightarrow \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 \Rightarrow kx_i^2 = mv_f^2$$

$$\Rightarrow v_f^2 = \frac{kx_i^2}{m} \Rightarrow v_f = \sqrt{\frac{kx_i^2}{m}} = \sqrt{\frac{(14)(0.032)^2}{0.011}} = 1.14161\frac{m}{s} = v_{i_{\parallel}}$$

This the final velocity at the end of the level surface which is also the initial velocity on the incline.



On the incline, we can not use Conservation of Mechanical Energy because there is work done by friction. We need to draw a free body diagram, break the force of gravity into its parallel and perpendicular components, redraw the free body diagram, sum the forces and use the uniformly accelerated motion equations.



$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg\cos\theta$$
$$\sum F_{\parallel} = -F_{g_{\parallel}} - F_{kf} = ma_{\parallel} \Rightarrow -mg\sin\theta - \mu_k F_N = -mg\sin\theta - \mu_k mg\cos\theta = ma_{\parallel}$$

Everybody brought mass to the party!!

$$\Rightarrow a_{\parallel} = -g\sin\theta - \mu_k g\cos\theta = -(9.81)\sin(25) - (0.3)(9.81)\cos(25) = -6.81315\frac{m}{s^2}$$

Now we can use a uniformly accelerated motion equation:

$$v_{f_{\parallel}}{}^2 = v_{i_{\parallel}}{}^2 + 2a_{\parallel}\Delta d_{\parallel} \Rightarrow 0 = v_{i_{\parallel}}{}^2 + 2a_{\parallel}\Delta d_{\parallel} \Rightarrow -v_{i_{\parallel}}{}^2 = 2a_{\parallel}\Delta d_{\parallel} \Rightarrow \Delta d_{\parallel} = \frac{-v_{i_{\parallel}}{}^2}{2a_{\parallel}}$$

$$\Rightarrow \Delta d_{\parallel} = \frac{-(1.14161)^2}{(2)(-6.81315)} = 0.095644m$$

$$\sin\theta = \frac{O}{H} = \frac{h_{max}}{\Delta d_{\parallel}} \Rightarrow h_{max} = \Delta d_{\parallel}\sin\theta = (0.095644)\sin(25) = 0.040421m \times \frac{100cm}{1m} \approx \boxed{4.0cm}$$