Flipping Physics Lecture Notes:

Introductory
Work due to Friction equals Change in Mechanical Energy
Problem

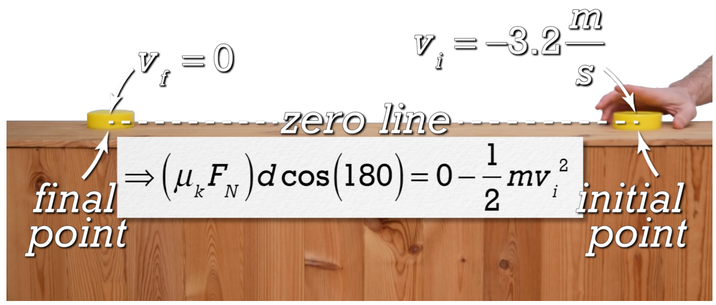
Problem: On a level surface, a street hockey puck is given an initial velocity of -3.2 m/s and slides to a stop. If the coefficient of kinetic friction between the puck and the surfaces is 0.60, how far did the puck slide?

Known values: $v_i = -3.2\frac{m}{s};\ v_f = 0;\ \mu_k = 0.60;\ \Delta x = ?$ (magnitude)

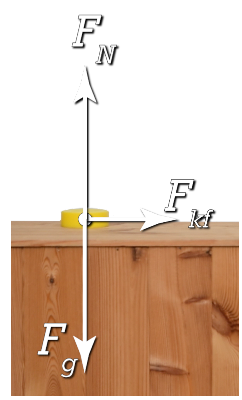
$$W_f = \Delta ME \Rightarrow F_{kf} d\cos\theta = ME_f - ME_i \Rightarrow \left(\mu_k F_N\right) d\cos\left(180\right) = 0 - \frac{1}{2} m v_i^2$$

The equation for force of kinetic friction is: $F_{kf} = \mu_k F_N$

The angle between the force of kinetic friction (to the right) and the displacement (to the left) is 180°. Set the initial point where the puck is released, the final point where it stops and the horizontal zero line at the height of the center of mass of the puck. The height initial and final are both zero, so the gravitational potential energy initial and final are both zero. There is no spring so the initial and final elastic potential energies are zero. The final velocity of the puck is zero, so the kinetic energy final is zero. The on the mechanical energy initial or final is the initial kinetic energy. Where did all the kinetic energy go? It is all converted to heat and sound as the puck slides.



We need the force normal, so we draw a free body diagram:



$$\sum F_y = F_N - F_g = ma_y = m\left(0\right) = 0 \Rightarrow F_N = F_g = mg$$

(and now back to the W_f equation)

$$\Rightarrow \mu_k \left(mg\right) d\left(-1\right) = -\frac{1}{2} m v_i^2 \Rightarrow \mu_k g d = \frac{1}{2} v_i^2$$

(everybody brought *negative* mass to the party!!)

$$\Rightarrow d = \frac{v_i^2}{2g\mu_k} = \frac{\left(-3.2\right)^2}{\left(2\right)\left(9.81\right)\left(0.60\right)} = 0.869861 \approx \boxed{0.87m}$$

Notice we get a positive answer for the distance the puck slid. That is because the work equation has the *magnitude* of the force and the displacement in it, therefore, when we solve for d, we only get the *magnitude* of the displacement of the object.

How I solved for the known variables in this problem:

From the video: $x_f = 32.2cm;\ x_i = 120.8cm;\ v_f = 0;\ \Delta t = 33 frames \times \frac{1 \sec}{60 frames} = \frac{33}{60} \sec$

$$\Delta x = x_f - x_i = 32.2 - 120.8 = -88.6cm \times \frac{1m}{100cm} = -0.886m$$

Now we can use a uniformly accelerated motion equation:

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t \Rightarrow \Delta x = \frac{1}{2}\left(v_i + 0\right)\Delta t \Rightarrow 2\Delta x = v_i \Delta t \Rightarrow v_i = \frac{2\Delta x}{\Delta t} = \frac{\left(2\right)\left(-0.886\right)}{\frac{33}{60}} = -3.221818\frac{m}{s}$$

In the problem we solved the work equation to get:

$$\mu_k g d = \frac{1}{2} v_i^2 \Rightarrow \mu_k = \frac{v_i^2}{2gd} = \frac{\left(-3.221818\right)^2}{\left(2\right)\left(9.81\right)\left(0.886\right)} = 0.597131$$

"Why did you use +0.886 meters and not -0.886 meters?" you ask. Remember (again) in the work equation, you use the magnitudes of the force and the displacement and "d" in the above equation came from the work equation.

Another note: The answer in the video is a distance of 0.87 meters and the measurement in the video was 0.89 meters; those two numbers are not the same. This is purely an issue of rounding. I felt my measured values were only good to two significant digits, which is why they are rounded to:

$v_i = -3.2\frac{m}{s}$ & $\mu_k = 0.60$. However, if you use the more precise values found when I originally solved for the variables: $v_i = -3.221818\frac{m}{s}$ & $\mu_k = 0.597131$, you get …

$$d = \frac{v_i^2}{2g\mu_k} = \frac{\left(-3.221818\right)^2}{\left(2\right)\left(9.81\right)\left(0.597131\right)} = 0.88600 \approx 0.886m$$

Which was what was measured in the video in the first place.