Flipping Physics Lecture Notes:

# Work due to Friction equals Change in Mechanical Energy Problem
by Billy

This is an alternate solution to a problem previously done by Billy.
http://www.flippingphysics.com/coe-incline-problem.html

Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μk between the block and the incline is 0.30, to what maximum height does the block slide?

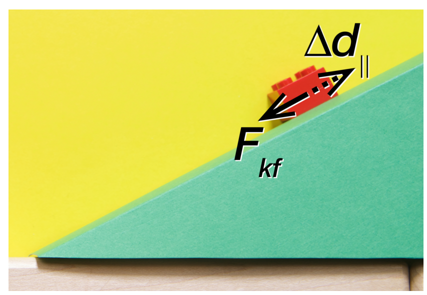
Last time we solved this problem using Conservation of Energy, Newton's Second Law and Uniformly Accelerated Motion. Mr.P asked me to show you how to solve the problem using the Work due to Friction equation.

We have the same knowns as last time:

$$k = 14\frac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{max} = ?$$

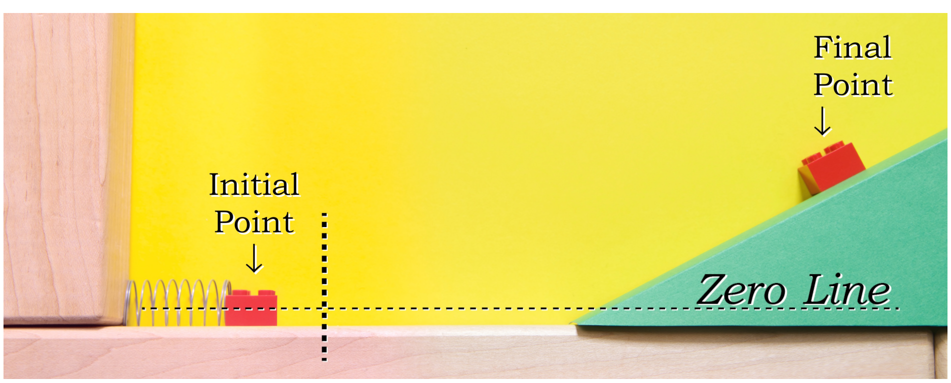
$$m = 11g \times \frac{1kg}{1000g} = 0.011kg\ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

$$W_f = \Delta ME \Rightarrow F_{kf} d\cos\theta = ME_f - ME_i \Rightarrow \mu_k F_N \Delta d_{\parallel}\cos(180) = mgh_f - \frac{1}{2}kx_i^2$$



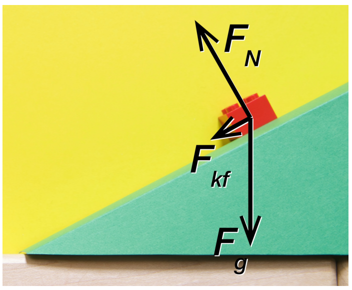

There is no friction on the level surface, so the only place there is Work due to Friction is on the incline. The angle between the Force of Kinetic Friction (down the incline) and the displacement of the block (up the incline) is 180 degrees.

Set the initial point where spring is compressed its maximum distance and the final point at the maximum height of the block. Set the horizontal zero line at the center of mass of the block when it is on the horizontal surface.



The initial velocity of the block is zero, so there is no initial kinetic energy. The initial height of the block is zero, so there is no initial gravitational potential energy. There is an initial compression of the spring, so there *is* kinetic energy initial.

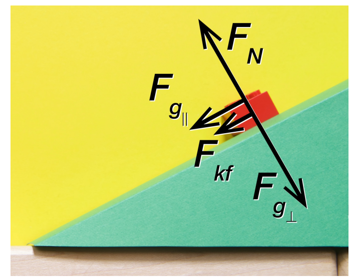
The final height of the block is not zero, so there *is* gravitational potential energy final. The final velocity of the block is zero, so there is no kinetic energy final. The block is not compressing the spring at the end, so there is no elastic potential energy final.




We need to find the force normal, so draw a free body diagram, break the force of gravity into its components, re-draw the free body diagram and sum the forces in the perpendicular direction.

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0$$

$$\Rightarrow F_N = F_{g_{\perp}} = mg\cos\theta$$




Going back to the Work due to Friction equation: $\Rightarrow \mu_k \left(mg\cos\theta\right)\Delta d_{\parallel}(-1) = mgh_f - \frac{1}{2}kx_i^2$

Draw a triangle to find the $\Delta d_{\parallel}$ in terms of $h_f$: $\sin\theta = \frac{O}{H} = \frac{h_f}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h_f}{\sin\theta}$


Substitute that back into the Work due to Friction equation:

$\Rightarrow -\mu_k mg\cos\theta\left(\frac{h_f}{\sin\theta}\right) = mgh_f - \frac{1}{2}kx_i^2$ and solve for $h_f$:

$\Rightarrow -\frac{\mu_k mg}{\tan\theta}\left(h_f\right) = mgh_f - \frac{1}{2}kx_i^2$ (because $\frac{\sin\theta}{\cos\theta} = \tan\theta$ )

$$\Rightarrow \frac{1}{2}kx_i^2 = mgh_f + \frac{\mu_k mgh_f}{\tan\theta} = mgh_f\left(1 + \frac{\mu_k}{\tan\theta}\right) \Rightarrow h_f = \frac{\frac{1}{2}kx_i^2}{mg\left(1 + \frac{\mu_k}{\tan\theta}\right)}$$

$$\Rightarrow h_f = \frac{\frac{1}{2}(14)(0.032)^2}{(0.011)(9.81)\left(1 + \frac{(0.3)}{\tan(25)}\right)} = \frac{0.007168}{(0.011)(9.81)(1.643352)} = 0.040421 \approx \boxed{0.040m = h_{max}}$$