Flipping Physics Lecture Notes:

An Introductory Kinetic Friction on an Incline Problem

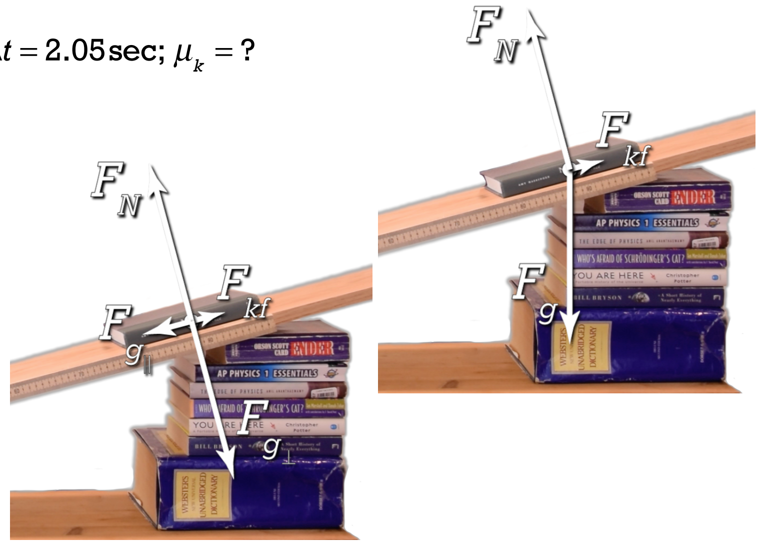
Example: You place a book on a 14° incline and then let go of the book. If the book takes 2.05 seconds to travel 0.78 meters, what is the coefficient of kinetic friction between the book and the incline?

Knowns: $\theta = 14°;\ v_{i_{\parallel}} = 0;\ \Delta d_{\parallel} = 0.78m;\ \Delta t = 2.05\sec;\ \mu_k = ?$

Draw the Free Body Diagram.

Break Force of Gravity into its components.

Redraw the Free Body Diagram.



$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg\cos\theta$$

$$\sum F_{\parallel} = F_{kf} - F_{g_{\parallel}} = ma_{\parallel} \Rightarrow \mu_k F_N - mg\sin\theta = ma_{\parallel} \Rightarrow \mu_k\left(mg\cos\theta\right) - mg\sin\theta = ma_{\parallel}$$

$$\Rightarrow \mu_k g\cos\theta - g\sin\theta = a_{\parallel} \Rightarrow \mu_k g\cos\theta = g\sin\theta + a_{\parallel} \Rightarrow \mu_k = \frac{g\sin\theta + a_{\parallel}}{g\cos\theta}$$

Now we need the acceleration in the parallel direction. Use a Uniformly Accelerated Motion equation.

$$\Delta d_{\parallel} = v_{i_{\parallel}}\Delta t + \frac{1}{2}a_{\parallel}\Delta t^2 = (0)\Delta t + \frac{1}{2}a_{\parallel}\Delta t^2 = \frac{1}{2}a_{\parallel}\Delta t^2 \Rightarrow 2\Delta d_{\parallel} = a_{\parallel}\Delta t^2 \Rightarrow a_{\parallel} = \frac{2\Delta d_{\parallel}}{\Delta t^2}$$

$$\Rightarrow 2\Delta d_{\parallel} = a_{\parallel}\Delta t^2 \Rightarrow a_{\parallel} = \frac{2\Delta d_{\parallel}}{\Delta t^2} = \frac{(2)(0.78)}{(2.05)^2} = 0.371208\frac{m}{s^2}$$

$$\mu_k = \frac{(9.81)\sin 14 + (0.371208)}{(9.81)\cos 14} = 0.288326 \approx 0.29$$

Note: This cannot be correct because previously we determined the coefficient of static friction between this book and this incline to be 0.27 and $\mu_k < \mu_s$. The mistake we made was that the displacement in the parallel direction is negative, which makes the acceleration negative, which changes our answer.

$$\Rightarrow a_{\parallel} = \frac{(2)(-0.78)}{(2.05)^2} = -0.371208\frac{m}{s^2}\ \ \&\ \ \mu_k = \frac{(9.81)\sin 14 + (-0.371208)}{(9.81)\cos 14} = 0.21030 \approx \boxed{0.21}$$