Flipping Physics Lecture Notes:

Calculating the Uncertainty of the Coefficient of Friction

Example: A book is resting on a board. One end of the board is slowly raised. The book starts to slide when the incline angle is 15°. What is the coefficient of static friction between the book and the incline?

As shown at http://www.flippingphysics.com/static-friction-incline.html the answer is:

$$\mu_s == \tan\theta = \tan(15) = 0.267949 \approx 0.27$$

The reality is that the measurements were as such:

Incline Angle (in degrees)	Tangent of Incline Angle and μ_s (6 significant digits)
13	0.230867
13	0.230867
14	0.248328
15	0.267949
15	0.267949
15	0.267949
16	0.286745
17	0.305731
17	0.305731
17	0.305731

The average value is $\mu_{s\ average} = 0.271855 \approx 0.27$

The standard deviation is $0.027527 \approx 0.03$
(Method #2 from this WikiHow shows how to calculate standard deviation. http://www.wikihow.com/Calculate-Uncertainty)

Therefore the coefficient of static friction is 0.27 with an uncertainty of plus or minus 0.03: $\boxed{\mu_s \approx 0.27 \pm 0.03}$

Note: I have illustrated the uncertainty with plus or minus 1 standard deviation, this means roughly 68%[1] of the measurements should fall within the range 0.24 – 0.30. If you would prefer, you can use 2 standard deviations instead. This would mean 95% of the measurements should fall within the range. Or, you could even use 3 standard deviations. This would mean 99.7% of the measurements should fall within the range. FYI: The symbol for standard deviation is typically a lowercase, Greek letter sigma, σ.

Average Value	Number of Standard Deviations (# of σ)	Uncertainty	Average Value with Uncertainty	Value Range	Predicted % of Measurements within the Range
0.27	1	$\sigma = 0.027527 \approx 0.03$	0.27 ± 0.03	$0.24 - 0.30$	68%
0.27	2	$2\sigma = (2)(0.027527) = 0.055054 \approx 0.06$	0.27 ± 0.06	$0.21 - 0.33$	95%
0.27	3	$3\sigma = (3)(0.027527) = 0.082582 \approx 0.08$	0.27 ± 0.08	$0.19 - 0.35$	99.7%

Also note: This way of calculating uncertainty is very useful and will be used more often as you get further into science and engineering. However, we will not be using it very often in our algebra based physics class.

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[1] http://www.statisticshowto.com/68-95-99-7-rule/