Flipping Physics Lecture Notes:

Instantaneous Power Delivered by a Car Engine - Example Problem

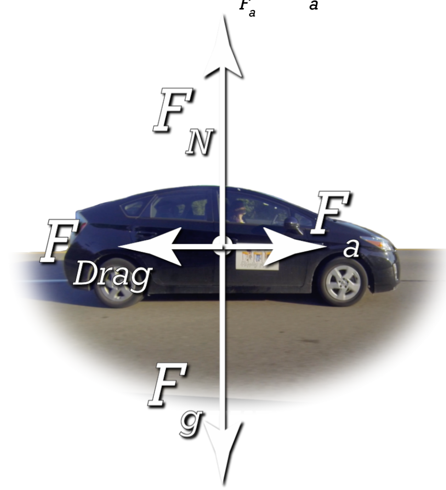
Example: A Toyota Prius is traveling at a constant velocity of 113 km/hr. If an average force of drag of 3.0 x 10^2 N acts on the car, what is the power developed by the engine in horsepower?

Knowns: $v = 113\frac{km}{hr} \times \frac{1000m}{1km} \times \frac{1hr}{3600\sec} = 31.3\bar{8}\frac{m}{s}; F_{Drag} = 300N; P_{by\ engine} = ?$

$$P = \frac{W}{\Delta t} = Fv\cos\theta$$

We can't use $P = \frac{W}{\Delta t}$ because we are solving for *instantaneous* power. We need to use $P_{F_a} = F_a v\cos\theta$ and use *instantaneous* velocity in the equation. We need F_a, and θ.

Draw Free Body Diagram:



$$\sum F_x = F_a - F_{Drag} = ma_x = m(0) = 0 \Rightarrow F_a = F_{Drag} = 300N$$

The force applied is to the right and the displacement is to the right, therefore the angle, θ, in the work equation is zero degrees. $\theta = 0^\circ$.

$$P_{F_a} = F_a v\cos\theta = (300)(31.3\bar{8})\cos(0) = 9416.\bar{6}\text{watts} \times \frac{1hp}{746watts} = 12.623 \approx \boxed{13\text{hp}}$$

Note: At 129 kilometers per hour …

$$v = 129\frac{km}{hr} \times \frac{1mi}{1.609km} = 80.174 \approx 80.2\frac{mi}{hr} \ \& \ v = 129\frac{km}{hr} \times \frac{1000m}{1km} \times \frac{1hr}{3600\sec} = 35.8\bar{3}\frac{m}{s}$$

$$P_{F_a} = F_a v\cos\theta = (390)(35.8\bar{3})\cos(0) = 13975\text{watts} \times \frac{1hp}{746watts} = 18.733 \approx 19\text{hp}$$

$$\frac{129\frac{km}{hr} - 113\frac{km}{hr}}{113\frac{km}{hr}} \times 100 = 14.159 \approx 14.2\% \ \& \ \frac{18.733hp - 12.623hp}{12.623hp} \times 100 = 48.406 \approx 48\%$$

In other words, in order to go 14% faster, the car consumes 48% more energy *every second*.

Force of Drag Calculation:

A standard equation for the Force of Drag is $F_{Drag}=\frac{1}{2}\rho v^2 DA$.

- ρ, is the density of the medium through which the object is moving. In our example, the density of air. In order to know the density of air, we need the air temperature, which was 72 °F.
  - $Temp_{air}=71^\circ F \Rightarrow T_{^\circ C}=\left(T_{^\circ F}-32\right)\left(\frac{5}{9}\right)=\left(71-32\right)\left(\frac{5}{9}\right)=21.\bar{6}\approx 22^\circ C$
  - According to The Engineering Toolbox[1], 1.2 kg/m^3 is a good approximation for the density of air at this temperature.
- v, is the instantaneous velocity of the car, $31.3\bar{8}\frac{m}{s}$.
- D, is the Drag Coefficient of the object, in this case the Sienna. According to EcoModder[2], the Drag Coefficient of my 2011 Toyota Prius is 0.25.
- A, is the Cross Sectional Area perpendicular to the direction of motion. This is the Frontal Area listed on the same EcoModder page. It is 21.6 square feet. Of course, we need it in square meters.

$$A=21.6ft^2\times\left(\frac{1m}{3.281ft}\right)^2=2.00651m^2$$

$$F_{Drag}=\frac{1}{2}\rho v^2 DA=\frac{1}{2}\left(1.2\right)\left(31.3\bar{8}\right)^2\left(0.25\right)\left(2.00651\right)=296.541\approx 3.0\times10^2\mathrm{N}\qquad @\left(113\frac{km}{hr}\right)$$

$$F_{Drag}=\frac{1}{2}\rho v^2 DA=\frac{1}{2}\left(1.2\right)\left(35.8\bar{3}\right)^2\left(0.25\right)\left(2.00651\right)=386.462\approx 390\mathrm{N}\qquad @\left(129\frac{km}{hr}\right)$$

For those of you who watched the average power video (http://www.flippingphysics.com/average-power.html), which used the same Prius, you might be wondering why that example had such a comparatively small force of drag at 8.0 N. The force is so much larger in this problem because the speed of the car is so much larger in this problem. The average speed in the average power problem was roughly 5 m/s, rather than roughly 31 m/s in this problem. Remember the speed is squared in the force of drag equation, which is why the force of drag is so much larger in this problem.

[1] http://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html
[2] http://ecomodder.com/wiki/index.php/Vehicle_Coefficient_of_Drag_List