Flipping Physics Lecture Notes:

AP Physics C: Work, Energy, and Power Review (Mechanics)
https://www.flippingphysics.com/apc-work-energy-power-review.html

- Work done by a constant force: $W = \vec{F} \bullet \Delta\vec{r} = F\Delta r\cos\theta$
  - Work is the dot product of Force and the displacement of the object.
    - The dot product is also called the scalar product, because it is a scalar.
    - $W \Rightarrow joules,\ J = N \cdot m = \left(\frac{kg \cdot m}{s^2}\right)m$
  - Example: $\vec{F} = \left[2.7\hat{i} - 3.1\hat{j}\right]N$ and $\Delta\vec{r} = 4.6\hat{i}\ m$ then **(include drawing)**

    $$W = \vec{F} \bullet \Delta\vec{r} = \left[2.7\hat{i} - 3.1\hat{j}\right] \bullet \left[4.6\hat{i} + 0\hat{j}\right] = (2.7)(4.6) + (-3.1)(0) = 12.42 \approx \boxed{12J}$$

- Work done by a non-constant force: $W = \int_{x_i}^{x_f} F_x\,dx$
  - This is a definite integral.
    - "Definite" simply means it has limits x_i and x_f.
    - Integral, or anti-derivative is the area "under" the curve.
      - Area "under" the curve specifically means the area between the curve and the horizontal axis where area above the horizontal axis is positive and area below the horizontal axis is negative.
- Notice we now have two different equations for work, one for work done by a constant force and one fore work done by a force that varies. This will happen very often in AP Physics C and you need to be careful to identify the difference.
- The force caused by a spring: $\vec{F}_s = -k\Delta\vec{x}$
  - k is the "spring constant" and is a measure of how much force is takes to compress or expand a spring per meter.
  - Δx is the displacement of the spring from equilibrium position (or rest position).
  - The negative means the force of the spring is opposite the direction of the displacement of the spring.
- $$W_{F_s} = \int_{x_i}^{x_f} F_s\,dx = \int_{x_i}^{x_f}(-kx)\,dx = \left(-\frac{kx^2}{2}\right)\Bigg]_{x_i}^{x_f} = \left(-\frac{kx_f^{\,2}}{2}\right) - \left(-\frac{kx_i^{\,2}}{2}\right)$$

  $$\Rightarrow W_{F_s} = -\left[\left(\frac{kx_f^{\,2}}{2}\right) - \left(\frac{kx_i^{\,2}}{2}\right)\right] = -\left[U_{ef} - U_{ei}\right] = -\Delta U_e \Rightarrow \boxed{W_{F_s} = -\Delta U_e}$$

  - We have defined elastic potential energy: $U_e = \frac{1}{2}kx^2$.
  - The above example shows the work done by the force of the spring equals the negative of the change in elastic potential energy of the spring.
- $\sum W = \int_{x_i}^{x_f} \sum F\,dx \Rightarrow \sum W = \Delta KE$ The Net Work – Kinetic Energy Theorem.
  - Derivation is here: http://www.flippingphysics.com/wnet-ke.html
  - This equation is *always* true.
  - This equation is where kinetic energy is defined: $KE = \frac{1}{2}mv^2$

- Gravitational Potential Energy in a constant gravitational field is: $U_g = mgh$
  - h is the "vertical height above the horizontal zero line" and you have to always identify the horizontal zero line.
  - If you prefer the equation from the AP sheet, it is: $\Delta U_g = mg\Delta h$
    - The AP equation is the "change in" gravitational potential energy.
- Energy can be neither created nor destroyed, so in a non-isolated system the change in energy of the system equals the sum of the energy transferred to or from the system: $\Delta E_{system} = \sum T$
  - If the system is isolated, no energy is transferred into or out of the system: $\Delta E_{system} = 0$
    - The change in energy of the system is the change in mechanical energy of the system plus the change in internal energy of the system. $\Delta ME + \Delta E_{internal} = 0$
    - The change in internal energy of the system is done by nonconservative forces or friction. In other words, the energy which is dissipated by friction goes into the system as internal energy. $\Delta E_{internal} = -W_{nc}$
      - A nonconservative force is a force where the work done by the force *is* dependent on the path taken by the object. Conservative forces are where the work done by the force is *not* dependent on the path taken by the object.
    - In other words: $\Delta ME - W_{nc} = 0 \Rightarrow W_{nc} = \Delta ME$ and because I don't know of any forces which are nonconservative which are not friction:
    - $W_{friction} = \Delta ME$ (is only true when there is no energy transferred into our out of the system.)
  - If the system is isolated and there is no work done by friction:
    - $W_{friction} = \Delta ME \Rightarrow 0 = \Delta ME = ME_f - ME_i \Rightarrow ME_i = ME_f$
    - We have conservation of mechanical energy.
    - Which is only true when the system is isolated and no work is done by friction.
- Whenever you use $W_{friction} = \Delta ME$ or $ME_i = ME_f$ you have to identify the initial point, the final point and the horizontal zero line.
- All forms of Mechanical Energy are in terms of joules, just like Work.
- Power is the rate at which work is done: $P_{average} = \frac{W}{\Delta t}$ & $P_{instantaneous} = \frac{dW}{dt}$
  - $P_{instantaneous} = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \bullet \Delta\vec{r}\right) = F \bullet \frac{d\Delta\vec{r}}{dt} = \vec{F} \bullet \vec{v}$
    - Note: Force must be constant to use this equation.
  - The equations for power on the AP sheet are: $P = \frac{dE}{dt}$ & $P = \vec{F} \bullet \vec{v}$
  - $P \Rightarrow Watts = \frac{J}{s}$ & $746 watts = 1 hp$
- Remember, every derivative is also an antiderivative (or an integral). For example:
  - $P = \frac{dW}{dt} \Rightarrow dW = P\,dt \Rightarrow \int_{W_i}^{W_f} dW = \int_{t_i}^{t_f} P\,dt \Rightarrow \Delta W = \int_{t_i}^{t_f} P\,dt$

- The equation which relates conservative forces and potential energy is: $F_x = -\frac{dU}{dx}$ (and it is *not* on the AP equation sheet.)
    - Aside: Much of the time when the phrase "conservative force" is used on the AP Exam, you need to use this equation.
    - For a spring: $F_s = -\frac{dU_e}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$
    - For gravity: $F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}\left(mgy\right) = -mg$ (Force of gravity is always down)
- Neutral Equilibrium is where the Potential Energy of the object remains constant regardless of position. For example, a ball rolling on a level surface.
- Stable Equilibrium is where the Potential Energy of the object increases as the position of the object moves away from the equilibrium position and therefore the object naturally returns to the equilibrium position. For example, a water bottle being tipped to the side.
- Unstable Equilibrium is where the Potential Energy of the object decreases as the position of the object moves away from the equilibrium position and therefore the object naturally moves away from the equilibrium position. For example, a marker being tipped to the side.