An object is in Free-Fall when the only force acting on the object is the Force of Gravity, however, we haven't defined Force much less the Force of Gravity, so, until we have defined the Force of Gravity, we have a slightly different definition.

An object is in Free-Fall when:
- It is not touching any other objects[*]
- There is no air resistance (it's in the vacuum we can breathe)

We are now in the vacuum that we can breathe and will be for the remainder of this class, unless otherwise stated.

Common Misconception: For some reason people think the word "fall" in Free-Fall means that the object must be going down. This is absolutely, not true. An object thrown upward is in Free-Fall from the moment it leaves the persons hand until it touches the ground.

When an object is in Free-Fall (on planet Earth):

$$a_y = -g = -9.81\frac{m}{s^2} = 9.81\frac{m}{s^2} down$$

-
- g is the acceleration due to gravity and on earth, g_{Earth} = 9.81 m/s^2.

The acceleration due to gravity, g, is different on different planets and moons. For example, g_{moon} = 1.6 m/s^2 which is roughly 1/6th of the acceleration due to gravity on the Earth.

It is *very* common for students at this point to assume that g is negative because a_y = -g. It is not. g is positive.

Please remember that. Please.

g is positive.

When an object is in Free-Fall we know the acceleration is constant, therefore:

An object in Free-Fall is an object experiencing Uniformly Accelerated Motion.
We can use the UAM equations and we know a_y = -9.81 m/s^2.

FYI: Mass is irrelevant. All objects, regardless of mass, will have the same acceleration.

When we look at the whole earth, the acceleration due to gravity will not be the same for each location. However, when we look at a specific location, the acceleration due to gravity is constant and therefore we can use the UAM equations. So little g is constant from a local perspective, however, when you look at it globally, the acceleration due to gravity varies from location to location.

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[*] We will get to some examples where we have two objects in free-fall together and those two objects will be touching. This is a good definition, for now.

Flipping Physics Lecture Notes:
Analyzing the Apollo 15 Feather and Hammer Drop
A Basic Introductory Free-Fall Problem

In 1971 Astronaut David Scott during the Apollo 15 mission on the moon dropped a feather and a hammer from the same height. The video shows that both objects fall at the same rate regardless of their different masses.

Knowns:

$$v_{iy} = 0;\; g_{moon} = 1.62\,\frac{m}{s^2};\; a_y = -g_{moon} = -1.62\,\frac{m}{s^2};\; \Delta t = 36\,frames \times \frac{1\sec}{29.97\,frames} = 1.201201\sec;\; h_i = ?$$

The acceleration is constant, so we can use the <u>U</u>niformly <u>A</u>ccelerated <u>M</u>otion Equations:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 = (0)\Delta t + \frac{1}{2}(-1.62)(1.201201)^2 = -1.16874 \approx -1.2m$$

For those of you still using English units:
$$\Delta y = -1.16874\,m \times \frac{3.281\,ft}{1m} = -3.8346\,ft \approx -3.8\,ft$$

So the hammer and feather were both dropped from about 1.2 meters or 3.8 feet off the surface of the moon.

Example Problem: Mr.p drops a medicine ball from a height of 2.0 m above the ground. (a) What is the velocity of the ball right before it strikes the ground? (b) How long did the ball fall?

Knowns: Δy = -2.0 m, a_y = -9.8 m/s^2, v_{iy} = 0, v_{fy} = ?, Δt = ?

Common mistake: The final velocity of the medicine ball is not zero. *After* the ball strikes the ground it's final velocity is zero, however, it isn't in free-fall anymore the moment it touches the ground.

Common mistake: Forgetting that the displacement is negative. It is negative because the ball is going down and down is negative. Or you could look at it this way: $\Delta y = y_f - y_i = 0 - 2 = -2.0m$

We know we can use the UAM equations because the acceleration is constant.

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = 0^2 + (2)(-9.81)(-2) \Rightarrow v_{fy} = \sqrt{(2)(-9.81)(-2)} = \pm 6.26418 \approx \boxed{-6.3\frac{m}{s}}$$

An equivalent answer would be 6.3 m/s down. However, please do not get overzealous and write -6.3 m/s down because, -6.3 m/s down = 6.3 m/s up, which is wrong.

Common mistake: Many of you will get v_{fy} = +6.26418 m/s because that is what your calculator says. Please be smarter than your calculator and remember, whenever you take the square root, that the answer could be positive or negative.

$$v_{fy} = v_{iy} + a_y\Delta t \Rightarrow -6.26418 = 0 + (-9.81)\Delta t \Rightarrow \Delta t = \frac{-6.26418}{-9.81} = 0.638551 \approx 0.64\sec$$

Possibly useful definitions:
Parallax (noun): the effect whereby the position or direction of an object appears to differ when viewed from different positions, e.g., through the viewfinder and the lens of a camera.
Eschew (verb): deliberately avoid using; abstain from.
Perspicacious (adjective): having or showing an ability to notice and understand things that are difficult or not obvious.
Overzealous (adjective): too zealous in attitude or behavior.
Zealous (adjective): having or showing zeal.
Zeal (noun): great energy or enthusiasm in pursuit of a cause or an objective.

Flipping Physics Lecture Notes:
Graphing the Drop of a Ball from 2.0 Meters
An Introductory Free-Fall Acceleration Problem

Example Problem: Mr.p drops a medicine ball from a height of 2.0 m above the ground. (a) What is the velocity of the ball right before it strikes the ground? (b) How long did the ball fall?

In the previous example we solved parts (a) and (b) and now we are going to draw the Position, Velocity and Acceleration as a function of time graphs.

Knowns: Δy = -2.0 m, a_y = -9.81 m/s^2, v_{iy} = 0;
Determined variables: v_{fy} = -6.26418 ≈ -6.3 m/s, Δt = 0.638551 ≈ 0.64 seconds
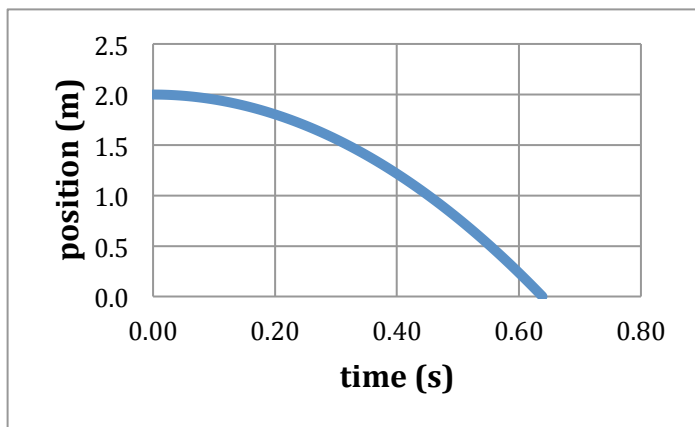
Acceleration vs. Time:
The easiest graph actually is acceleration vs. time. We know it is an object in free-fall; therefore its acceleration is constant and has a value of -9.81 m/s^2. So we draw a horizontal line at -9.81 m/s^2.

Velocity vs. Time:
Next, the slope of a velocity vs. time graph is acceleration; therefore the velocity vs. time graph has a constant slope of -9.81 m/s^2. We also know the initial velocity is zero. That completes our velocity vs. time graph.
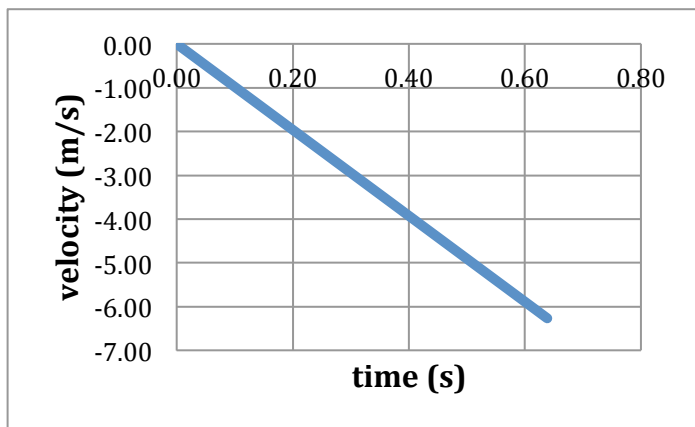
Position vs. Time:
The initial position is 2.0 meters.

We also know the slope of the position vs. time graph is velocity and the velocity at zero seconds is zero, therefore the initial slope of the position vs. time graph needs to be zero.

As time increases, the velocity gets more and more negative and therefore the slope of the position vs. time graph needs to get more and more negative. So the graph will be a curve the goes down.

We also know that the position of zero corresponds to a time of 0.638551 seconds, so we can approximate our position vs. time graph.

(we could go through and figure out several points for a better approximation, however, this is good enough for today.)
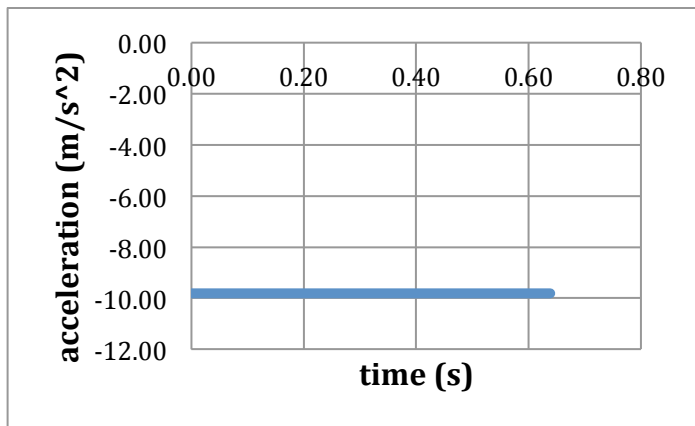
Flipping Physics Lecture Notes:
Throwing a Ball up to 2.0 Meters & Proving the Velocity at the Top is Zero
An Introductory Free-Fall Acceleration Problem

We have already done this example problem and drawn the graphs of position, velocity and acceleration as a function of time.

Previous Example Problem: Mr.p drops a medicine ball from a height of 2.0 m above the ground. (a) What is the velocity of the ball right before it strikes the ground? (b) How long did the ball fall?

Knowns: Δy = -2.0 m, a_y = -9.81 m/s^2, v_{iy} = 0;
Determined variables: v_{fy} = -6.26418 $\approx$ -6.3 m/s, Δt = 0.638551 $\approx$ 0.64 seconds

Previous graphs:
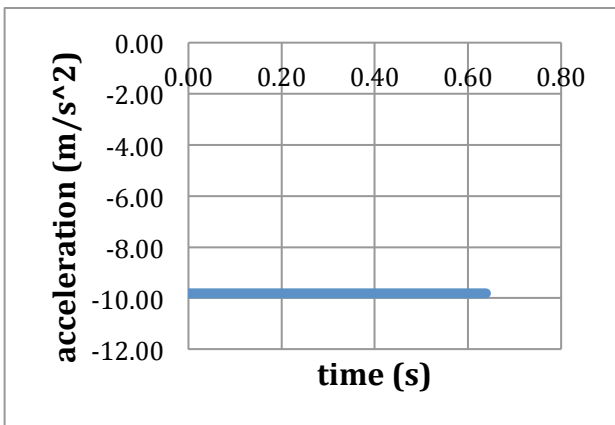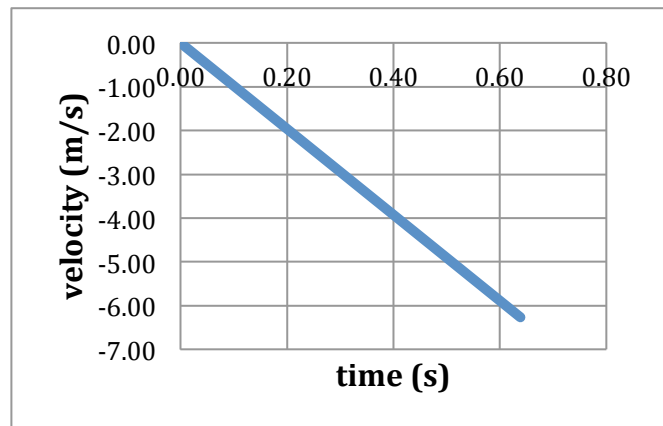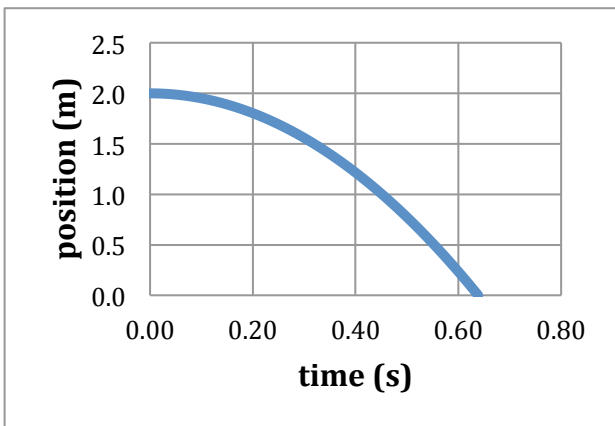
Note: Our solution to this problem is based on having solved the previous problem and is meant to make comparisons between the two. This is why we can start with the graphs to solve this problem.

(this point starts all the new stuff for this problem)

Now we have the problem: Mr.p throws a medicine ball upward and catches it again at the same height that he threw it. If the maximum height the ball achieves above where he threw it is 2.0 meters, how long was the ball not in mr.p's hands?

Acceleration vs. Time Graph:
Just like last time, the easiest graph to start with is the acceleration graph because the value is constant at -9.81 m/s^2 because the medicine ball is in free-fall. Therefore the acceleration vs. time graph has a horizontal line at -9.81 m/s^2.
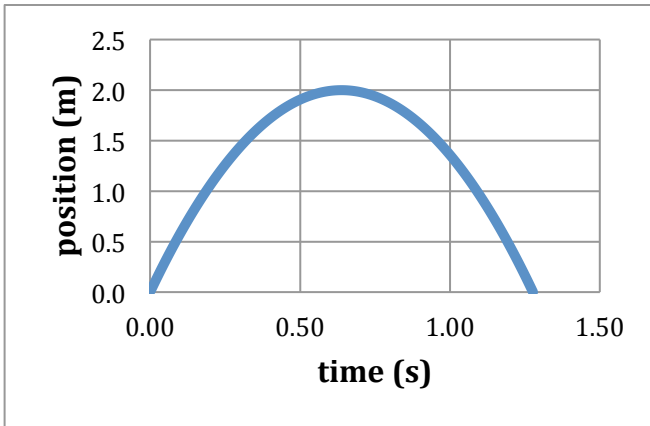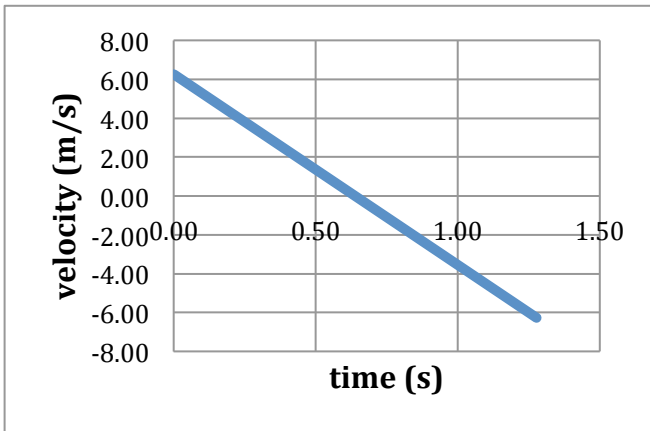
Velocity vs. Time Graph:

We know that the velocity vs. time graph has a slope of -9.81 m/s², however, we need to know where it starts and ends. I won't prove it right now, however, I know it will start at 6.26418 m/s and end at -6.26418 m/s. We'll know why in a moment.

Position vs. Time Graph:

With regards to position as a function of time, we don't know the initial height, so we can just pick an arbitrary initial point of zero meters. And we know it ends at the same height or zero meters. We also know it has a maximum height of 2.0 meters, so in the middle it will have a maximum value of 2.0 meters.

We know the initial slope of the position vs. time graph needs to be about 6.3 m/s and the final slope needs to be -6.3 m/s. (the slope of the position vs. time graph is velocity) This means that the slope will constantly decrease from 6.3 m/s to -6.3 m/s. So the position vs. time graph is concave down and symmetrical.

But what does this mean that the velocity is zero right in the middle of the graph? It means that *the velocity is zero at the very top of the balls path*. The velocity is positive on the way up and negative on the way down, so at the very top it must be zero. You can also see it in the position as a function of time graph, the slope of the line at the very top is zero, and therefore the velocity at the top is zero.
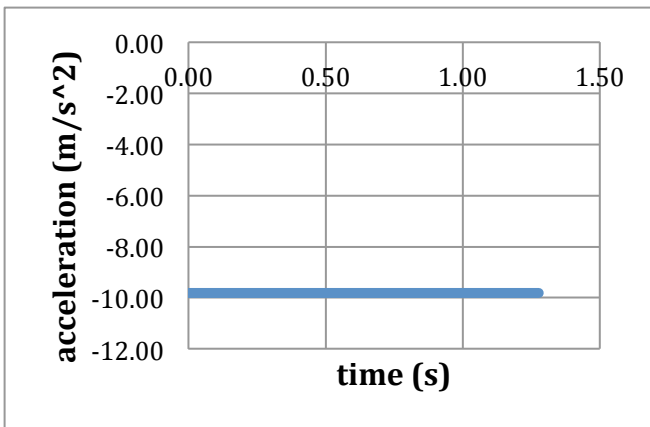
Notice that the 2nd half of these graphs are *exactly* the same as the graphs we drew in the previous lecture. Therefore we can conclude that the 2nd half took 0.638551 seconds and due to symmetry, the first half will also take 0.638551 seconds. Therefore the total time is:

$$\Delta t_{total} = \Delta t_{up} + \Delta t_{down} = 0.638551 + 0.638551$$

$$\Rightarrow \Delta t_{total} = (2)(0.638551) = 1.277102 \approx \boxed{1.3\,\text{sec}}$$

Where does the initial velocity of +6.26418 m/s come from? Well we know, for the whole event:

$$\Delta t_t = 1.277102\,\text{sec}; \; v_{fy} = -6.26418\frac{m}{s}; \; a_y = -9.81\frac{m}{s^2}; \; \Delta y_t = 0; \; v_{iy} = ?$$

$$v_{fy}^{\,2} = v_{iy}^{\,2} + 2a_y\Delta y_t \Rightarrow v_{fy}^{\,2} = v_{iy}^{\,2} + (2)(-9.81)(0) \Rightarrow v_{iy} = \sqrt{v_{fy}^{\,2}} \Rightarrow v_{iy} = \pm v_{fy} = -(-6.26418) \approx \boxed{+6.3\frac{m}{s}}$$
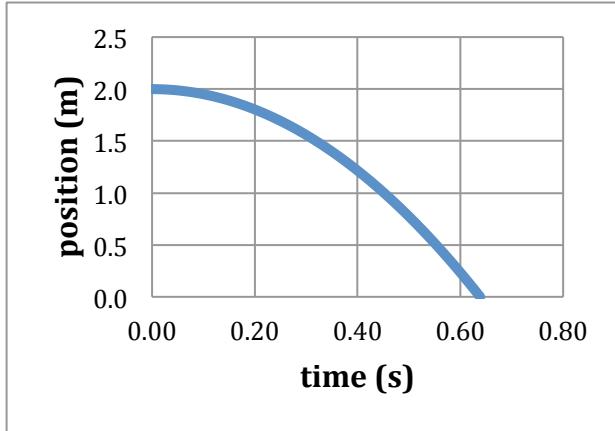
We know we must use the positive answer, because the ball is moving upward.

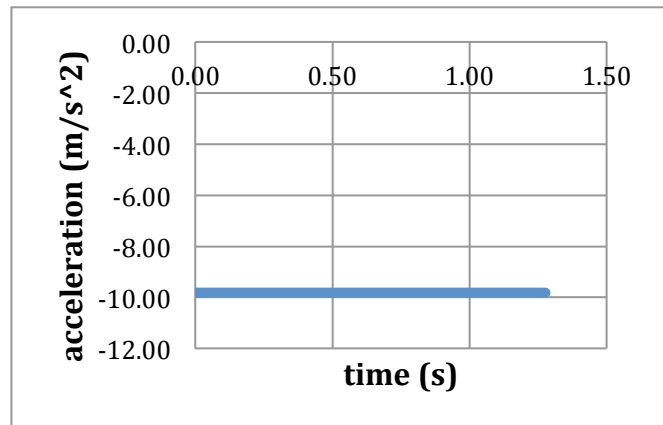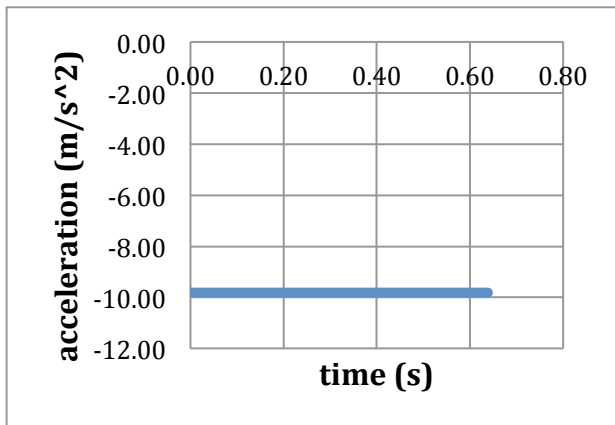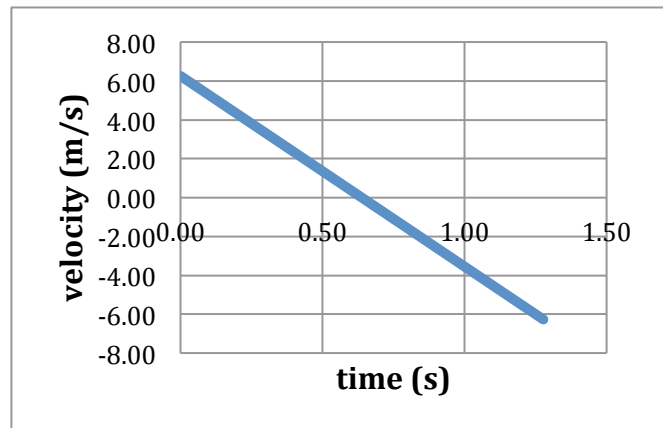Note: The times will *not* be the same going up and down if the Δy ≠ 0.

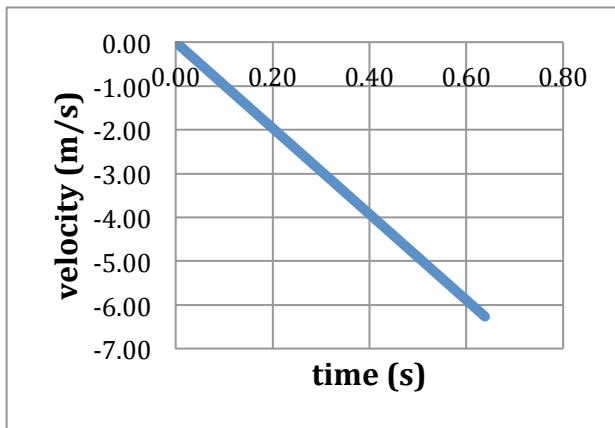Graphs for Dropping a Ball from 2.0 Meters

Graphs for Throwing a Ball up to 2.0 Meters
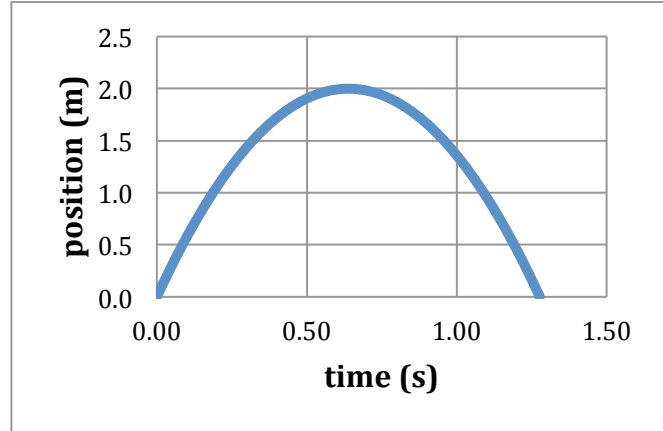
Shown in the video:
Because the 2nd half of the Throw graphs is exactly the same as the Drop graphs, the two events are the same.

If you go backwards in time on the Drop graphs, you get the 1st half of the Throw graphs, so if we play the Drop video backwards and then forwards, you will get the Throw graph.

Position as a function of Time Graph for Dropping a Ball from 2.0 Meters
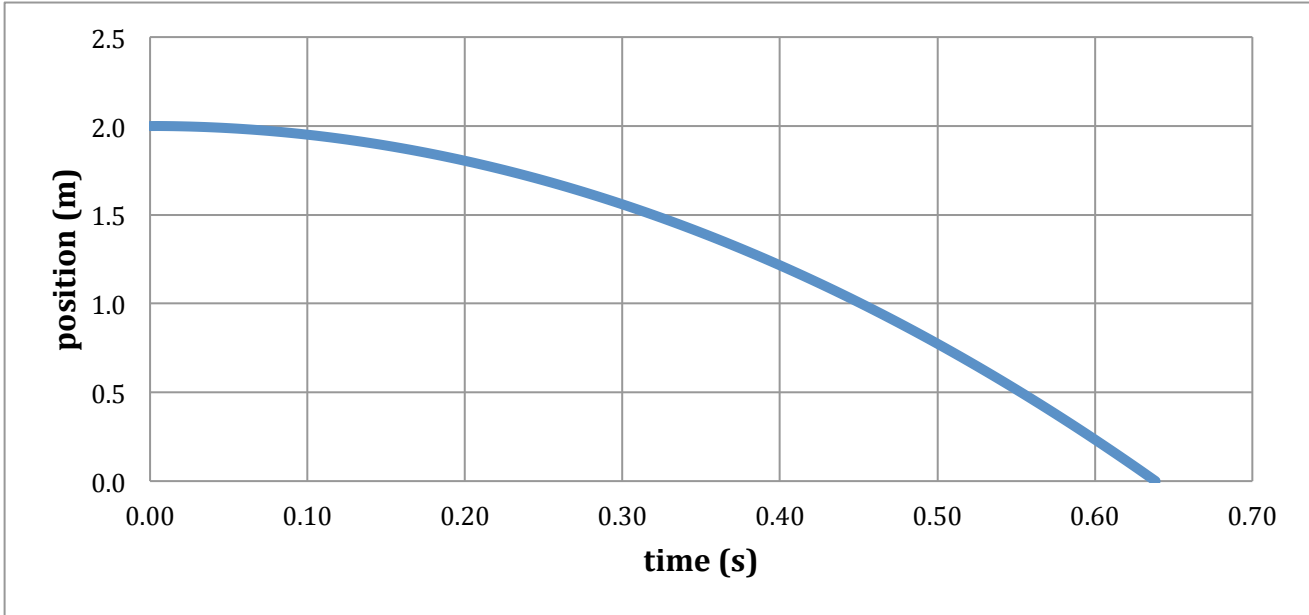
Stop Motion Photography showing the position of the ball as a function of time.

Yep, they are the same.

There are several common misconceptions or mistakes students make when it comes to free-fall problems. These are all problems where the object is flying through the vacuum that you can breathe and isn't touching anything else and therefore:
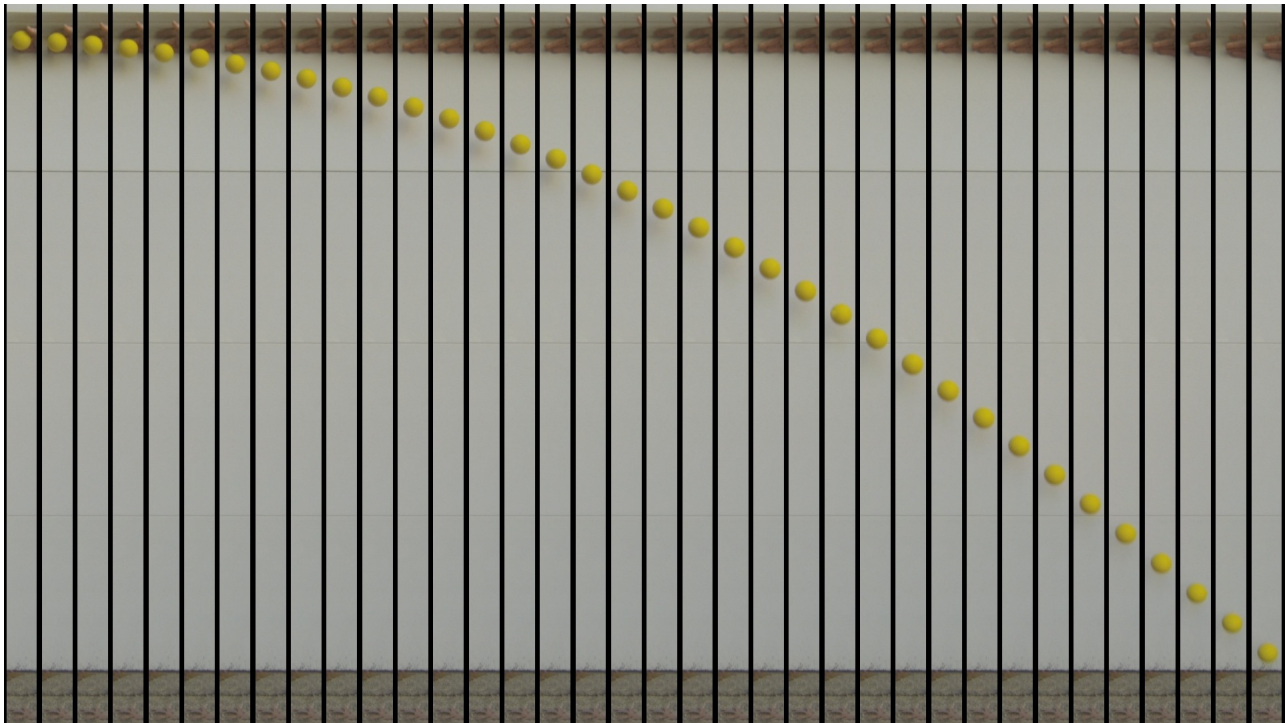
$$g_{Earth} = 9.81 \frac{m}{s^2} \text{ and } a_y = -g = -9.81 \frac{m}{s^2} = 9.81 \frac{m}{s^2} Down$$

1st, if the object is going up, students often assume that the acceleration must be positive because the velocity is positive. This is not true, the object's velocity is decreasing, and therefore the acceleration is opposite the direction of the velocity and is therefore down and negative.

Sometimes students tell me that the initial velocity for an object being thrown upward is zero. That doesn't work. If the initial velocity in the y-direction is not positive, the object will not go up. In fact, the initial velocity in the y-direction must be positive for the object to move upward.

Also, students will often think that the object will accelerate faster if you throw it down rather than drop it. It will be moving faster, yes, however, because it is an object in free-fall, it will not *accelerate* faster than 9.81m/s^2 down.

Next one isn't a mistake students make, it is just a reminder that the velocity at the top, in the y-direction, is zero. The velocity is positive on the way up and negative on the way down, so it must pass through zero at the top. Therefore, the object stops at the top and has a velocity, in the y-direction, of zero. Which leads us to the last one.

Lastly, students sometimes think that, because the object's velocity at the top is zero, then it's acceleration must also be zero. This is not true. The equation for acceleration is: $a = \frac{\Delta v}{\Delta t}$ therefore, if the acceleration at the top were zero, then Δv would equal zero. (We are assuming time doesn't stop and therefore Δt ≠ 0, which seems pretty reasonable). If Δv = 0 then the velocity would not change and would continue to be zero and the object would stop at the top and float in midair. I think we can all agree that, in this universe, that does not happen. So it is illogical that the acceleration at the top would be zero. So, again, it is an object in free-fall and therefore:

$$g_{Earth} = 9.81 \frac{m}{s^2} \text{ and } a_y = -g = -9.81 \frac{m}{s^2} = 9.81 \frac{m}{s^2} Down$$

1) Object going up. a_y =-9.81 m/s^2 (acceleration still negative).
2) Object thrown upward, $v_{iy} > 0$.
3) Object thrown downward. a_y =-9.81 m/s^2 (Yes, it is moving faster. No it doesn't *accelerate* more).
4) v_{top} in y-direction = 0
5) a_{top} = -9.81 m/s^2 (again, in the y-direction and even though v_{top} = 0).

Example Problem: Mr.p throws a ball straight up and lets it fall to the ground. If the ball leaves his hand 124 cm above the ground and it lands on the ground 1.14 seconds later, to what maximum height did the ball go above the ground?

The key to understanding this problem is that you have to split the event into two parts. We will call them Part 1: Going Up and Part 2: Falling Down. However, you actually have to start by looking at the whole event first.

Total Event: Knowns: $\Delta y_t = -124\,cm \times \dfrac{1m}{100\,cm} = -1.24m$; $a_y = -9.81\dfrac{m}{s^2}$; $\Delta t_t = 1.14\,\text{sec}$; $v_{ti} = ?$

$$\Delta y_t = v_{ti}\Delta t_t + \frac{1}{2}a_y\Delta t_t^{\,2} \Rightarrow -1.24 = v_{ti}(1.14) + \frac{1}{2}(-9.81)(1.14)^2 = 1.14v_{ti} - 6.374538$$

$$\Rightarrow 1.14v_{ti} = -1.24 + 6.374538 = 5.134538 \Rightarrow v_{ti} = \frac{5.134538}{1.14} = 4.503981\frac{m}{s} = v_{1i}$$

(The initial velocity for the Total Event is the same as the initial velocity for Part 1: Going Up)

Now we just look at Part 1: Going Up: Knowns: $v_{1i} = 4.503981\dfrac{m}{s}$; $a_y = -9.81\dfrac{m}{s^2}$; $v_{top} = v_{1f} = 0$; $\Delta y_1 = ?$

$$v_{1f}^{\,2} = v_{1i}^{\,2} + 2a_y\Delta y_1 \Rightarrow (0)^2 = v_{1i}^{\,2} + 2a_y\Delta y_1 \Rightarrow -\left(v_{1i}^{\,2}\right) = 2a_y\Delta y_1$$

$$\Rightarrow \Delta y_1 = \frac{-\left(v_{1i}^{\,2}\right)}{2a_y} = \frac{-(4.503981)^2}{(2)(-9.81)} = 1.033937m$$

We have the displacement while going up, however, we need to add the initial height to that to get the maximum height the ball went above the ground.

$$h_{max} = h_{initial} + \Delta y_1 = 1.24 + 1.033937 = 2.273937 \approx \boxed{2.27m}$$

Flipping Physics Lecture Notes:
Dropping Dictionaries Doesn't Defy Gravity, Duh!
Video Proof of the Mass Independence of the Acceleration due to Gravity

In the video I drop 1, then 2, then 3, then 4 books all from the same height and they all take the exact same amount of time to strike the ground. Therefore, all their accelerations in the y-direction are the same.

All four examples share these same Uniformly Accelerated Motion Variables:

$v_i = 0$; $\Delta t = 0.64$ seconds, $\Delta y = -2.0$ meters.

If these three variables are the same, then so are their accelerations in the y direction:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 = (0)\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow \Delta y = \frac{1}{2}a_y\Delta t^2 \Rightarrow a_y = \frac{2\Delta y}{\Delta t^2} = \frac{(2)(-2)}{(0.64)^2} = -9.76563 \approx -9.8\frac{m}{s^2}$$

Each set of books, regardless of mass, has an acceleration in the y-direction of -9.8 m/s^2.

Oh, and if you arrange them in a looping sequence, you can dance to it.

Possibly Useful Definitions:
Pummel (verb): to strike repeatedly, typically with the fists.
    (yeah, so I didn't use it in its traditional sense, however, my wife really can pummel me in Scrabble)
Dilapidated (adjective): in a state of disrepair or ruin as a result of age or neglect.
Voila (exclamation): there it is; there you are. ORIGIN French *voilà.*

Problem: You are wearing your rocket pack (m_{total} = 75 kg) that accelerates you upward at a constant 10.5 m/s^2. While preparing to take pictures of the beautiful view, you drop your camera 5.0 seconds after liftoff. 5.0 seconds after you drop the camera, (a) what is the camera's velocity and (b) how far are you from the camera?

The mass is entirely useless in this problem. This reiterates that mass does not influence the free-fall acceleration of an object.

Identify that there are both two parts to this problem and two different objects. Part 1: You and the camera are together accelerating upward before you drop the camera. Part 2: You and the camera are separated after dropping the camera.

For Part 1) Both you and the camera are moving upward together for 5 seconds.

$$v_{1i} = 0 \, ; \; a_1 = 10.5 \frac{m}{s^2} \, ; \; \Delta t_1 = 5.0s \, ; \; v_{1f} = v_{2ic} = v_{2iu} = ?$$

(the velocity at the end of part 1 is the same as the initial velocity for part 2 for both you (subscript "u") and the camera (subscript "c") Because the acceleration is constant, we can use the Uniformly Accelerated Motion Equations.

$$v_{2fc} = v_{2ic} + a_{2c}\Delta t_2 = 52.5 + (-9.81)(5) = 3.45 \approx 3.5 \frac{m}{s}$$

Part 2, just the camera: $v_{2ic} = 52.5 \frac{m}{s} \, ; \; \Delta t_2 = 5.0s \, ; \; a_{2c} = -9.81 \frac{m}{s^2}$ (the camera is now in free-fall)

$$v_{2fc} = v_{2ic} + a_{2c}\Delta t_2 = 52.5 + (-9.81)(5) = 3.45 \approx \boxed{3.4 \frac{m}{s}}$$

(We have just discovered that, because the velocity is positive that the camera is still moving *up*.)

Now we need to figure out how far apart the two are, continuing with Part 2, just the camera:

$$\Delta y_{2c} = \frac{1}{2}\left(v_{2fc} + v_{2ic}\right)\Delta t_2 = \frac{1}{2}(3.45 + 52.5)(5) = 139.875m$$

Now part 2, just you: $v_{2iu} = 52.5 \frac{m}{s} \, ; \; a_{2u} = -9.81 \frac{m}{s^2}$ (rockets still firing); $\Delta t_2 = 5.0s \, ; \; \Delta y_{2u} = ?$

$$\Delta y_{2u} = v_{2iu}\Delta t_2 + \frac{1}{2}a_{2u}\left(\Delta t_2\right)^2 = (52.5)(5) + \frac{1}{2}(10.5)(5)^2 = 393.75m$$

Lastly, we know that your displacement for part 2 is equal to the displacement for part 2 for the camera plus the distance between you and the camera (or the answer to part (b)). In other words …

$$\Delta y_{2u} = \Delta y_{2c} + (b) \Rightarrow (b) = \Delta y_{2u} - \Delta y_{2c} = 393.75 - 139.875 = 253.875 \approx \boxed{250m}$$

Possibly Useful Definition:
arcane (adjective): understood by few; mysterious or secret