- $\vec{\tau}=\vec{r}\times\vec{F}\neq\vec{F}\times\vec{r}$
  - Torque is the cross product (also called the vector product) of $\vec{r}$ & $\vec{F}$.
    - Torque is a vector!
  - $\vec{r}$ is the position vector from the axis of rotation to the location of the force, $\vec{F}$.
  - Magnitude of torque → $\tau = rF\sin\theta$
  - The order does matter! ($\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$)
  - Cross product is the area of the parallelogram with sides $\vec{r}$ & $\vec{F}$.
- In case you forgot how to do the cross product. Example: $\vec{A}=-\hat{i}+\hat{j}-2\hat{k}$ & $\vec{B}=2\hat{i}-3\hat{j}+4\hat{k}$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -2\\ 2 & -3 & 4\end{vmatrix}=\begin{vmatrix}1 & -2\\ -3 & 4\end{vmatrix}\hat{i}-\begin{vmatrix}-1 & -2\\ 2 & 4\end{vmatrix}\hat{j}+\begin{vmatrix}-1 & 1\\ 2 & -3\end{vmatrix}\hat{k}$$

$$\Rightarrow \vec{A}\times\vec{B}=\left[(1)(4)-(-2)(-3)\right]\hat{i}-\left[(-1)(4)-(-2)(2)\right]\hat{j}+\left[(-1)(-3)-(1)(2)\right]\hat{k}$$

$$\Rightarrow \vec{A}\times\vec{B}=\left[4-6\right]\hat{i}-\left[-4+4\right]\hat{j}+\left[3-2\right]\hat{k}=\boxed{-2\hat{i}+\hat{k}}$$

- An object is in *Translational* Equilibrium if the net force acting on it equals zero, which means the object is not accelerating: $\sum\vec{F}=0=m\vec{a}\Rightarrow\vec{a}=0$
- An object is in *Rotational* Equilibrium if the net torque acting on it equals zero, which means the object is not *angularly* accelerating: $\sum\vec{\tau}=0=I\vec{\alpha}\Rightarrow\vec{\alpha}=0$ (must identify axis of rotation)
  - This means the object is either not rotating or has a constant angular velocity.
  - If an object is in translational equilibrium and in rotational equilibrium about *one* axis of rotation, then the object is in rotational equilibrium about *any* axis of rotation.
- $\vec{L}$ is Angular Momentum and it is a vector!
  - $\sum\vec{F}=m\vec{a}\Rightarrow\sum\vec{\tau}=I\vec{\alpha}$ & $\sum\vec{F}=\frac{d\vec{p}}{dt}\Rightarrow\sum\vec{\tau}=\frac{d\vec{L}}{dt}$
- For a *particle* or any object which is *not rotating*:
  - Just like torque, we have a cross product equation for angular momentum: $\vec{L}=\vec{r}\times\vec{p}$
    - r is the position vector from the axis of rotation to the location of the center of mass of the moving object.
    - And a magnitude equation for angular momentum: $L=rmv\sin\theta$
      - With this equation, need to use Right Hand Rule to find direction.
  - Yes, a particle or a rigid object which is not rotating can have an angular momentum!
- For a *rigid object with shape*: $\vec{L}=I\vec{\omega}$
  - Units for angular momentum: $\vec{L}=I\vec{\omega}\Rightarrow\left(kg\cdot m^2\right)\left(\frac{rad}{s}\right)=\frac{kg\cdot m^2\cdot rad}{s}=\frac{kg\cdot m^2}{s}$
- Derivation of conservation of *linear* momentum: $\sum\vec{F}_{external}=\frac{d\vec{p}}{dt}=0\Rightarrow\sum\vec{p}_i=\sum\vec{p}_f$
- Derivation of conservation of *angular* momentum: $\sum\vec{\tau}_{external}=\frac{d\vec{L}}{dt}=0\Rightarrow\sum\vec{L}_i=\sum\vec{L}_f$

- Note the similarities between the two, please.
- Remember net torque requires the axis of rotation to be identified, which means the axis of rotation needs to be identified for conservation of angular momentum

- Conservation of Angular Momentum Example: A piece of gum with mass, m, and velocity, v, is spat at a solid cylinder of mass, M, radius, R, and moment of inertia $\frac{1}{2}MR^2$. The cylinder is on a horizontal axis through its center of mass and is initially at rest. The line of action of the gum is located horizontally a height, y, above the axis of the cylinder. If the gum sticks to the cylinder, what is the final angular velocity of the gum/cylinder system? The **Drawing!!**
  - Gum knowns: $m = m_g, v = v_{gi}$ Cylinder knowns: $M = m_c, \omega_{ic} = 0, R, I_c = \frac{1}{2}m_c R^2$
  - Solving for $\omega_f = ?$ (will be the same for both gum and cylinder)
  - Know angular momentum is conserved because: $\sum \vec{\tau}_{external} = \frac{d\vec{L}}{dt} = 0$
  - $\sum \vec{L}_i = \sum \vec{L}_f \Rightarrow \vec{L}_{gi} + \vec{L}_{ci} = \vec{L}_{gf} + \vec{L}_{cf} \Rightarrow r_{gi} m_g v_{gi} \sin\theta_{gi} + 0 = r_{gf} m_g v_{gf} \sin\theta_{gf} + I_c \omega_f$
  - $\sin\theta_{gi} = \frac{O}{H} = \frac{y}{r_{gi}} \Rightarrow y = r_{gi}\sin\theta_{gi} \Rightarrow y m_g v_{gi} = r_{gf} m_g v_{gf} \sin\theta_{gf} + I_c \omega_f$
  - $v_{gf} = v_t = R\omega_f \Rightarrow y m_g v_{gi} = R m_g R \omega_f \sin 90 + \frac{1}{2} m_c R^2 \omega_f$
  - $\Rightarrow y m_g v_{gi} = R^2 \omega_f \left(m_g + \frac{m_c}{2} \right) \Rightarrow \boxed{\omega_f = \frac{y m_g v_{gi}}{R^2 \left(m_g + \frac{m_c}{2} \right)}}$

FYI: Sawdog, one of my Quality Control Team members, pointed out that, after colliding with the cylinder, the gum is moving in a circle, so it's angular momentum can be described using $I_g \omega_f$. More specifically: $\vec{L}_{gf} = I_g \omega_f = \left(m_g r_g^{\ 2} \right) \omega_f = m_g R^2 \omega_f$ It's a slightly different solution that results in the same answer.