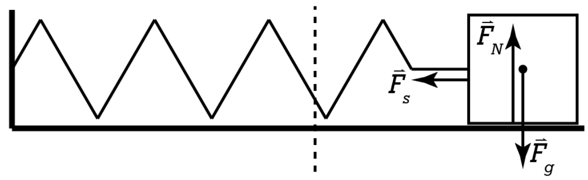
Flipping Physics Lecture Notes:

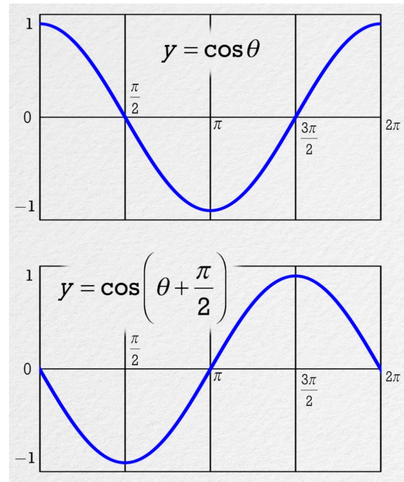
AP Physics C: Simple Harmonic Motion Review (Mechanics)
https://www.flippingphysics.com/apc-simple-harmonic-motion-review.html

- An object is in Simple Harmonic Motion if the acceleration of the object is proportional to the object's displacement from an equilibrium position and that acceleration is directed toward the equilibrium position. $a \propto \Delta x$
- For example: A horizontal mass-spring system on a frictionless surface has the following free body diagram:

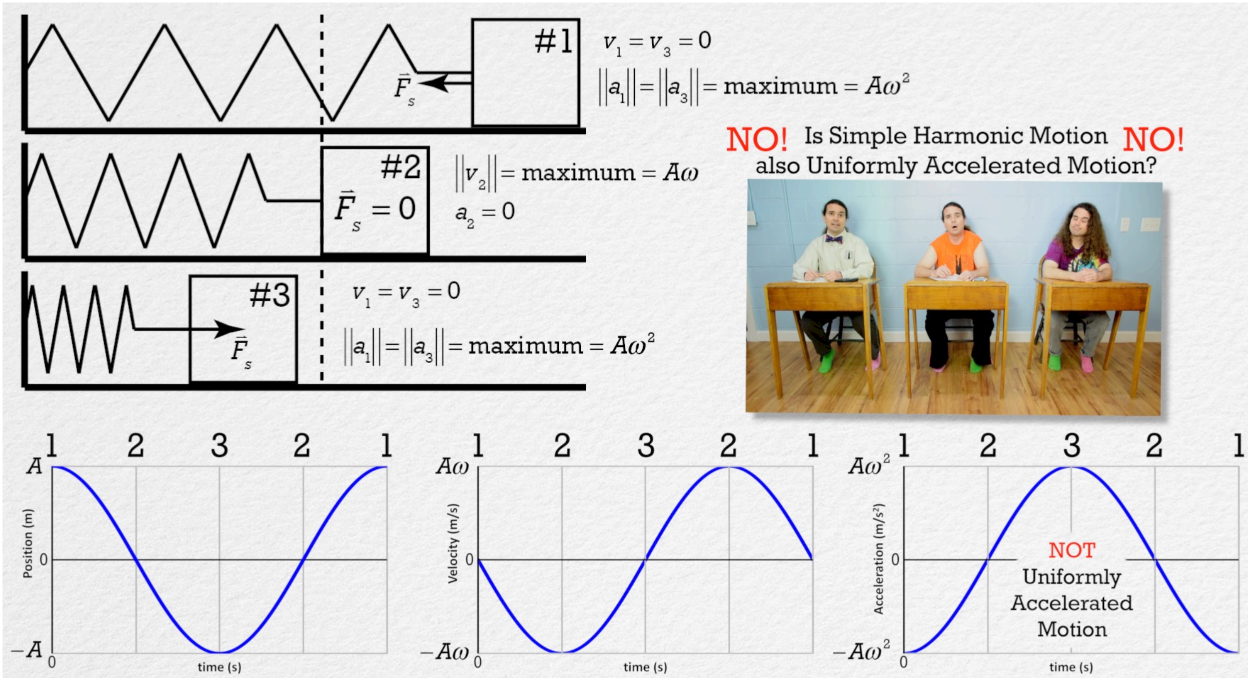


  - $\sum F_x = -F_s = ma_x \Rightarrow -kx = ma_x \Rightarrow a_x = -\frac{k}{m}x$
  - Amplitude, A, is defined as the maximum distance from equilibrium position. Therefore:
    - $a_{\max} = \frac{k}{m}A$
  - Note: $a = \frac{dv}{dt}$ & $v = \frac{dx}{dt} \Rightarrow a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$
  - Therefore: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$
  - Let $\frac{k}{m} = \omega^2$ where ω is called the angular frequency
  - Therefore: $\frac{d^2x}{dt^2} = -\omega^2 x$
    - This is the condition for simple harmonic motion.
    - This equation is *not* on the AP equation sheet. **Memorize It!!**
  - Note: $\omega = \sqrt{\frac{k}{m}} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$ (The period of a mass-spring system)
  - Period of a pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ (know how to derive)
  - $T = \frac{1}{f}$ & $\omega = \frac{2\pi}{T} = 2\pi f \Rightarrow \omega = 2\pi f$
    - Frequency, f, is the number of cycles an object goes through per second.
    - Angular frequency and frequency are related, $\omega = 2\pi f$, however, they are not the same.

- One equation that satisfies the condition for Simple Harmonic Motion is: $x(t) = A\cos(\omega t + \phi)$
  - This equation *is* on the AP physics equation sheet, however, the equations for velocity and acceleration in simple harmonic motion are **not**.
  - Have to use angles in radians in this equation.
  - ϕ or "phi" is the "phase constant" or the "phase shift" of the wave. For example:
    - $y = \cos\left(\theta + \frac{\pi}{2}\right)$ is "phase shifted" to the left from $y = \cos\theta$ by $\frac{\pi}{2}$ radians.



- $v = \frac{dx}{dt} = \frac{d}{dt}\left(A\cos(\omega t + \phi)\right) = A\left(-\sin(\omega t + \phi)\right)(\omega)$
  - $\Rightarrow v(t) = A\frac{d}{dt}\left(\cos(\omega t + \phi)\right) = A\left(-\sin(\omega t + \phi)\right)\left[\frac{d}{dt}(\omega t + \phi)\right]$
  - $\Rightarrow v(t) = -A\omega\sin(\omega t + \phi)$
  - & $v_{max} = A\omega$
- $a = \frac{dv}{dt} = \frac{d}{dt}\left(-A\omega\sin(\omega t + \phi)\right) = -A\omega\frac{d}{dt}\left(\sin(\omega t + \phi)\right) = -A\omega\cos(\omega t + \phi)\left[\frac{d}{dt}(\omega t + \phi)\right]$
  - $\Rightarrow a = -A\omega\left(\cos(\omega t + \phi)\right)(\omega) \Rightarrow a(t) = -A\omega^2\cos(\omega t + \phi)$
  - & $a_{max} = A\omega^2$
- $\Rightarrow a(t) = -\omega^2\left(A\cos(\omega t + \phi)\right) = -\omega^2 x(t) \Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x$



- Simple Harmonic Motion is **NOT** Uniformly Accelerated Motion
- Total mechanical energy in Simple Harmonic Motion:
  - $ME_{total} = \frac{1}{2}kA^2 = \frac{1}{2}m\left(v_{\max}\right)^2$