Flipping Physics Lecture Notes:

Introduction to Tangential Acceleration with Record Player Example Problem

When an object is moving in a circle…

- it travels a linear distance which is called arc length.
  - $s = r\Delta\theta$
- it has a linear velocity which is called tangential velocity.
  - $v_t = r\omega$
- it has a linear acceleration which is called tangential acceleration.
  - $a_t = r\alpha$
- You must use radians in all three of these equations!

Example Problem: A record player is plugged in and uniformly accelerates to 45 revolutions per minute in 0.85 seconds. Mints are located 3.0 cm, 8.0 cm, and 13.0 cm from the center of the record. What is the magnitude of the tangential acceleration of each mint?

Knowns: $\omega_i = 0; \omega_f = 45\frac{rev}{\min}\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1\min}{60\sec}\right) = 1.5\pi\frac{rad}{s}; \Delta t = 0.85\text{sec};$

$r_1 = 3.0cm; r_2 = 8.0cm; r_3 = 13.0cm; a_t = ?\ (\text{each})$

First we need to solve for the angular acceleration of the record player and therefore each of the mints:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{1.5\pi - 0}{0.85} = 5.54399\frac{rad}{s^2}$$

Now we can solve for the tangential acceleration of each mint:

$$a_{t1} = r_1\alpha = (3)(5.54399) = 16.63196\frac{cm\cdot rad}{s^2} \approx \boxed{17\frac{cm}{s^2}}$$

$$a_{t2} = r_2\alpha = (8)(5.54399) = 44.35190 \approx \boxed{44\frac{cm}{s^2}}$$

$$a_{t3} = r_3\alpha = (13)(5.54399) = 72.07183 \approx \boxed{72\frac{cm}{s^2}}$$

Tangential velocity and tangential acceleration are by definition tangent to the circle through which the object is moving, that is what the word tangential means. This also means the tangential velocity and acceleration are perpendicular to the radius of the circle.

If you understand the derivative, you can see the relationship between the arc length, tangential velocity, and tangential acceleration equations:

$$s = r\Delta\theta \Rightarrow \frac{d}{dt}(s = r\Delta\theta) \Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v_t = r\omega$$

$$v_t = r\omega \Rightarrow \frac{d}{dt}(v_t = r\omega) \Rightarrow \frac{dv_t}{dt} = r\frac{d\omega}{dt} \Rightarrow a_t = r\alpha$$