Free Body Diagrams:

Notice how all free body diagrams are different at different bucket locations. However, force of gravity is always down and force of tension is always in.

Let's start by analyzing when the bucket is at the bottom.

$$\sum F_{in} = F_T - F_g = ma_c$$

$$\Rightarrow F_T - mg = mr\omega^2$$

$$\Rightarrow F_T = mg + mr\omega^2$$
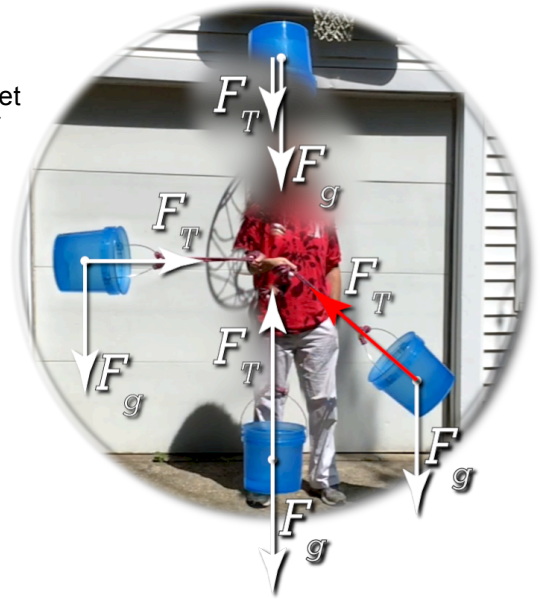
A couple of things to notice:
1) The centripetal force is Force of Tension minus Force of Gravity.
2) The faster the bucket spins the larger the force of tension and therefore …
3) The faster the bucket spins the larger the centripetal force necessary to keep the bucket moving in circular motion.

If we knew the radius of the path, the mass and angular speed of the bucket and water we could solve for the force of tension.♥ We could do this at any bucket location: Top, Bottom, Side, etc. Notice at a location which is not the bottom, top, or side, we would need to break the force of gravity into its components in the in-direction and the tangential-direction, just like we did with the toy car and the curved hill. (http://www.flippingphysics.com/car-hill-force-normal.html) But, we don't have a numbers dependency, so let's stop here and not have any numbers in this lesson. ☺

♥ For those of you taking AP Physics 1, be aware I have yet to see a problem on the AP Physics 1 exam that has the bucket in a position other than the top, bottom, or side.