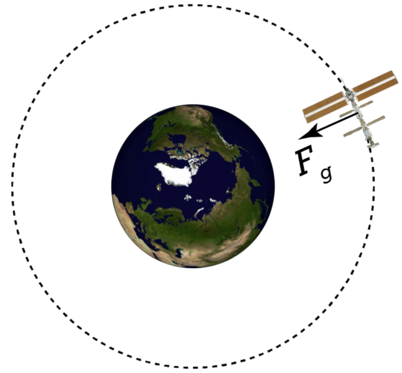
Flipping Physics Lecture Notes:

Altitude of Geosynchronous Orbit
(a.k.a. Geostationary Orbit)



Example: What is the altitude of a satellite in geosynchronous orbit?

Knowns: $m_{Earth} = 5.9723\ x\ 10^{24}\,kg;\ R_{Earth-Equatorial} = 6.378\ x\ 10^{6}\,m$ ♥

Geostationary orbit is where the object orbiting the Earth is always above the same spot on the Earth. Note: In order for this to work, the orbiting object must be above the Earth's equator.♦

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_1m_2}{r^2} = mr\omega^2$$

Please note the error in the video: Geosynchronous orbit and geostationary orbit are not the same. Geostationary orbit is a special case of geosynchronous orbit. A geosynchronous orbit simply has the same 24 hour period as the Earth, however, it is inclined relative to the equator and traces out an ellipse in the sky as seen from the Earth. Sorry they are incorrectly identified as the same in the video. Thank you to Dan Burns @kilroi22 and Christopher Becke @BeckePhysics for the correction!

Because the satellite is always above the same spot on the Earth, the satellite has the same angular velocity as the earth. In other words, the satellite goes through one revolution every 24 hours.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi rad}{24hr} \times \frac{1hr}{3600s} = 7.27221 \times 10^{-5}\,\frac{rad}{s}$$

We need the angular velocity in terms of seconds. This is because G is in $\frac{N \cdot m^2}{s^2}$.

$r = R_{Earth} + Altitude$; Distance from center of mass of Earth to center of mass of satellite.

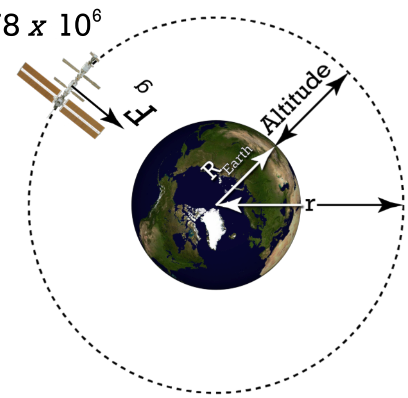
Note: r is the same on both sides of the equation.

$$\Rightarrow \frac{Gm_sm_E}{r^2} = m_sr\omega^2 \Rightarrow \frac{Gm_E}{r^2} = r\omega^2 \Rightarrow \frac{Gm_E}{r^3} = \omega^2 \Rightarrow r^3 = \frac{Gm_E}{\omega^2} \Rightarrow r = \sqrt[3]{\frac{Gm_E}{\omega^2}}$$

$$r = \sqrt[3]{\frac{\left(6.67 \times 10^{-11}\right)\left(5.9723\ x\ 10^{24}\right)}{\left(7.27221 \times 10^{-5}\right)^2}} = 4.22323 \times 10^{7}\,m$$

$$r = R_{Earth} + Altitude \Rightarrow Altitude = r - R_{Earth} = 4.22323 \times 10^{7} - 6.378\ x\ 10^{6}$$

$$\Rightarrow r = 3.58543 \times 10^{7} \approx \boxed{3.59 \times 10^{7}\,m}$$



Note: NASA lists the altitude of geosynchronous orbit as 35,900 km.♣

$35{,}900km \times \frac{1000m}{1km} = 3.59 \times 10^{7}\,m$; Yep, we got it right!

♥ https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html

♦ Just so you know, this means all satellite dishes on planet Earth that communicate with a geostationary satellite will point towards a location above the equator.

♣ https://www.nasa.gov/multimedia/imagegallery/image_feature_388.html