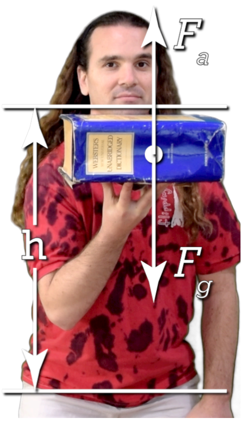
Flipping Physics Lecture Notes:

Deriving the Binding Energy of a Planet

All objects on planet Earth are gravitationally bound to it. Jump up and you will discover it is very difficult to leave the planet. However, if you are given enough mechanical energy, you will be able to leave the planet. The minimum amount of energy or work necessary to completely remove an object from another object is called the “binding energy.” One can also determine the “binding energy” of atoms and atomic particles. In this lesson we are determining the binding energy of a planet.



In order to determine an object’s binding energy, let’s start by analyzing lifting a book at a constant velocity a short, vertical distance, h:

$$\sum F_y = F_a - F_g = ma_y = m(0) = 0 \Rightarrow F_a = F_g = mg$$

Because the book is moving at a constant velocity, the force applied to the book and the force of gravity acting on the book are equal in magnitude.

$$W = Fd\cos\theta \Rightarrow W_{F_a} = F_a d\cos\theta = (mg)(h)\cos(0) = mgh$$

The work done by the force applied equals mass times acceleration due to gravity times h which is, in this case, the change in height of the book. Notice mgh is the value for the gravitational potential energy in a constant gravitational field. Also, θ is the angle between the direction of the force applied and the displacement of the book, and is therefore zero degrees.

If we set the horizontal zero line at the initial height of the book, then $h_i = 0$ and $h_f = h$ then:

$$\Delta PE_g = PE_{gf} - PE_{gi} = mgh_f - mgh_i = mgh - mg(0) = mgh$$

Therefore the work done by the force applied on the book equals the change in gravitational potential energy of the book: $W_{F_a} = \Delta PE_g$

Remember the binding energy of a planet is the minimum amount of energy or work necessary to completely remove an object from the planet. In this case the gravitational potential energy we need to use is $U_g = -\frac{Gm_1m_2}{r}$ because the gravitational field is not constant.

$$W_{F_a} = \Delta U_g = U_{gf} - U_{gi} = -\frac{Gm_om_E}{r_f} - \left(-\frac{Gm_om_E}{r_i}\right) = -\frac{Gm_om_E}{\infty} + \frac{Gm_om_E}{R_E} = \boxed{\frac{Gm_om_E}{R_E}}$$

Because the object is being completely removed from the planet, the final position is infinitely far away from the planet, $r_f = \infty$, and anything divided by infinity equals zero.