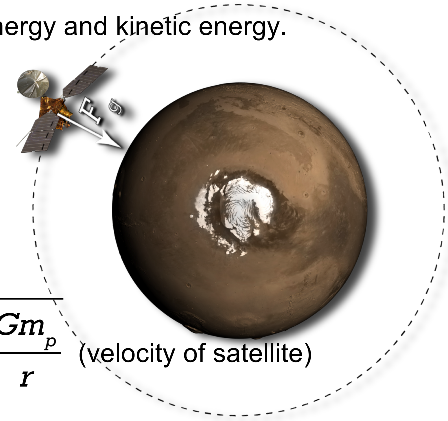
Flipping Physics Lecture Notes:

Mechanical Energy of a Satellite

A satellite is an object in orbit which has both gravitational potential energy and kinetic energy.

$$ME_{total} = U_g + KE = -\frac{Gm_s m_p}{r} + \frac{1}{2} m_s v_s^{2}$$



In order to simplify this expression we need to identify that the only force acting on the satellite is the force of gravity which is directed toward the center of mass of the planet. Therefore we can sum the forces in the in-direction on the satellite.

$$\sum F_{in} = F_g = ma_c \Rightarrow \frac{Gm_s m_p}{r^2} = m_s \frac{v_s^{2}}{r} \Rightarrow \frac{Gm_p}{r} = v_s^{2} \Rightarrow v_s = \sqrt{\frac{Gm_p}{r}}$$ (velocity of satellite)

(everybody brought the mass of the satellite divided by the radius to the party)

Substitute $v_s^{2} = \frac{Gm_p}{r}$ into the ME_{total} equation:

$$ME_{total} = -\frac{Gm_s m_p}{r} + \frac{1}{2} m_s \frac{Gm_p}{r} = \frac{Gm_s m_p}{r}\left(-1 + \frac{1}{2}\right) = \boxed{\left(-\frac{1}{2}\right)\frac{Gm_s m_p}{r}}$$

That's right, the total mechanical energy of a satellite equals half the universal gravitational potential energy between the satellite and the planet: $ME_{total} = \left(-\frac{1}{2}\right)\frac{Gm_s m_p}{r} = \frac{1}{2}\left(-\frac{Gm_s m_p}{r}\right) = \frac{1}{2}U_g$

Realize, because Universal Gravitational Mechanical energy is always negative, the Total Mechanical Energy is still negative.

I can't help but point out this out about the escape velocity we determined in a previous lesson:

$$v_{escape} = \sqrt{\frac{2Gm_p}{r}} = \sqrt{2}\sqrt{\frac{Gm_p}{r}} = \left(\sqrt{2}\right) v_{satellite}$$

That is correct, the escape velocity equals $\sqrt{2}$ times the satellite velocity.
I don't know why that is, or why it is interesting, however, it is interesting.

FYI: The free body diagram picture shows NASA's Mars Reconnaissance Orbiter in orbit around Mars.