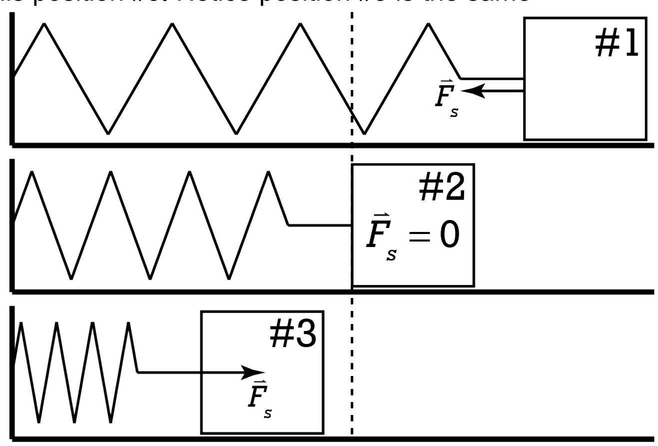
Flipping Physics Lecture Notes:

Simple Harmonic Motion Introduction via a Horizontal Mass-Spring System

A Horizontal Mass-Spring System is where a mass is attached to a spring, oriented horizontally, and then placed on a frictionless surface. When the mass is at rest, the spring will be at the equilibrium or rest position. This is the vertical line shown in the diagram below. We can then pull the mass to the right and hold it there. Let's call this position #1. Notice at position #1 the force of the spring, $\vec{F}_{s1}$, is to the left because the displacement of the spring from equilibrium position, $\vec{x}_1$, is to the right. We know this because of Hooke's Law, $\vec{F}_s = -k\vec{x}$.

When we let go of the mass the spring force will accelerate the mass to the left, the mass will pass through the equilibrium position, which we can call position #2. After passing through rest position, the mass will pause to the left of the equilibrium position. Let's call this position #3. Notice position #3 is the same distance from position #2 as position #1.



At position #2, the displacement from equilibrium position, $\vec{x}_2$, is zero. Therefore, according to Hooke's Law, the force of the spring, $\vec{F}_{s2}$, is also equal to zero.

At position #3, the displacement from rest position, $\vec{x}_3$, is to the left. Therefore, according to Hooke's Law, the force of the spring, $\vec{F}_{s3}$, is to the right.

In the absence of friction, the spring will continue to move back and forth through these positions like this: 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1 … and it will never stop.

This is called Simple Harmonic Motion. There are two requirements for the force that causes simple harmonic motion:

1) It must be a Restoring Force: A force that is always towards equilibrium.
    a. The spring force is a restoring force because it is always directed toward rest position and therefore will always accelerate the mass toward equilibrium position.
2) The force must be proportional to displacement from equilibrium position.
    a. According to Hooke's Law, $\vec{F}_s = -k\vec{x}$, the spring force is proportional to displacement from equilibrium position. In other words, the larger the displacement from equilibrium position, the larger the spring force.