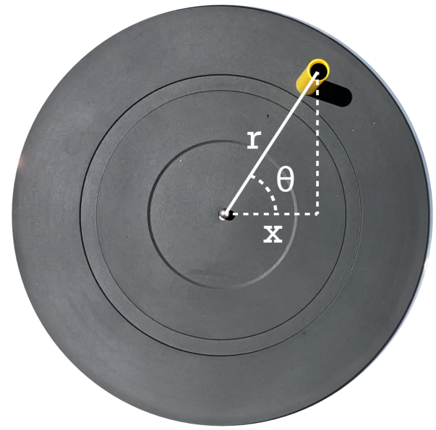
Flipping Physics Lecture Notes:

Simple Harmonic Motion – Position Equation Derivation

Circular motion, when viewed from the side, is simple harmonic motion. We can use this fact to derive an equation for the position of an object in simple harmonic motion.



- r is radius of the circular motion.
- x is the position of the cap in the x-direction, assuming the center of the turntable is the center of our coordinate system.
- θ is the angular displacement of the cap from an initial position where the cap was at its extreme position to the right.

$$\cos\theta = \frac{A}{H} = \frac{x}{r} \Rightarrow x = r\cos\theta \text{ \&}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\theta - 0}{t - 0} = \frac{\theta}{t} \Rightarrow \theta = \omega t$$

- Assuming we let $\theta_i = 0; t_i = 0; \theta_f = \theta; t_f = t$

$$x = r\cos\theta = r\cos(\omega t) \text{ \& } \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f \qquad \text{remember: } f = \frac{1}{T}$$

Therefore $\theta = \omega t = 2\pi f t$ and $x = r\cos\theta = r\cos(\omega t) = r\cos(2\pi f t)$

Identifying that the maximum displacement from equilibrium position is the amplitude, A, is also r in the position equation. Therefore: $x(t) = A\cos(2\pi f t)$

Notice this is an equation which can be used to describe an object oscillating in simple harmonic motion. The equation could also be: $x(t) = A\sin(2\pi f t)$ or even $x(t) = A\cos(2\pi f t + \phi)$.

- ϕ is the phase constant and phase shifts the sine and cosine wave along the horizontal axis. Realize ϕ is not in the AP Physics 1 curriculum, however, it is very useful.
- For example: $x(t) = A\cos(2\pi f t) = A\sin\left(2\pi f t + \frac{\pi}{2}\right)$

Some useful points:

- θ was in radians in our derivation, therefore angles in the equations for simple harmonic motion are in radians and your calculator needs to be in radians when using these equations.
- ω is angular frequency which is ***not*** the same as frequency, f.
  - $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$
- Yes, $T = \frac{2\pi}{\omega}$, is on the AP Physics equation sheets, however, you are much better served to remember and understand its derivation.