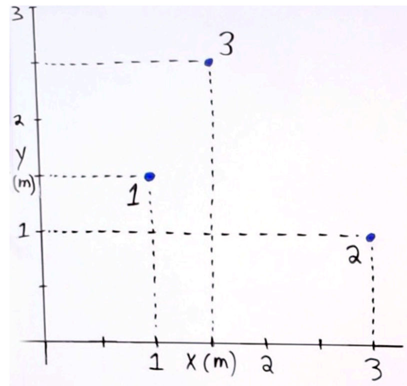
Flipping Physics Lecture Notes:

Calculating the Center of Mass of a System of Particles

The equation for the position of the center of mass of a system of particles is:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}$$



Where "m" is the mass of each object and "x" is the distance each object is from a zero reference point. The ellipses (…) mean you add as many expressions as you have objects in the system.

Example: Three point objects are located at various locations on a Cartesian coordinate system. Mass 1, with a mass of 1.1 kg, is located at (1.0,1.5) m. Mass 2, with a mass of 3.4 kg, is located at (3.0,1.0) m. Mass 3, with a mass of 1.3 kg, is located at (1.5,2.5) m. Where is the center of mass of the three-object system?

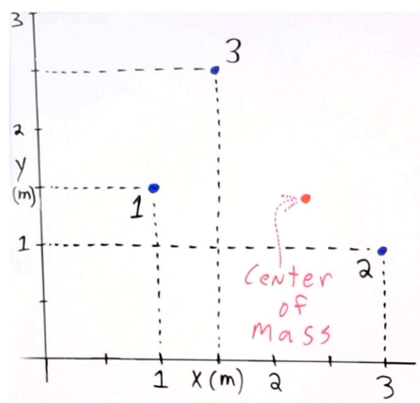
Knowns:

$$m_1 = 1.1kg;\ r_1 = (1.0,1.5)m;\ m_2 = 3.4kg;\ r_2 = (3.0,1.0)m;\ m_3 = 1.3kg;\ r_3 = (1.5,2.5)m;\ r_{cm} = ?$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(1.1)(1) + (3.4)(3) + (1.3)(1.5)}{1.1 + 3.4 + 1.3} = 2.28448 \approx 2.3\text{m}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(1.1)(1.5) + (3.4)(1) + (1.3)(2.5)}{1.1 + 3.4 + 1.3} = 1.43103 \approx 1.4\text{m}$$

$$r_{cm} \approx (2.3,1.4)\text{m}$$



Note: The Center of Mass is different than the Centroid, which is the geometric center, or where the center of mass would be if all of the masses were the same.

$$x_{avg} = \frac{x_1 + x_2 + x_3}{3} = \frac{1 + 3 + 1.5}{3} = 1.8\overline{3} \approx 1.8m$$

$$y_{avg} = \frac{y_1 + y_2 + y_3}{3} = \frac{1.5 + 1 + 2.5}{3} = 1.\overline{6} \approx 1.7m$$

$$r_{centroid} \approx (1.8,1.7)m$$