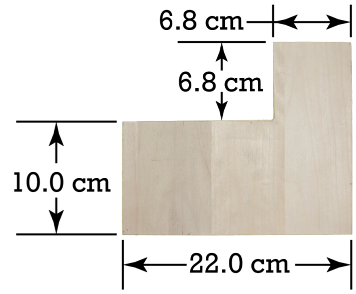
Center of Mass of an Irregular Object

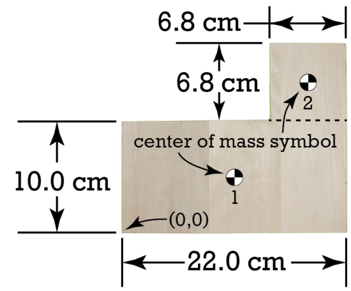
Where is the center of mass of an "L" shaped, constant density, constant thickness block with the dimensions shown in the illustration?



Set the zero, zero location or origin at the lower leftmost corner of the block. Split the block into symmetrical shapes with known centers of mass locations.

Piece 1 is 22.0 by 10.0 cm and piece 2 is 6.8 by 6.8 cm. Both pieces have, due to symmetry and constant density, a center of mass at their geometric center:

$$r_1 = (11,5)cm \;\&\; r_2 = \left(22 - \frac{6.8}{2}, 10 + \frac{6.8}{2}\right) = (18.6, 13.4)cm$$



We can use the equation for center of mass of a system of particles to determine the center of mass of the "L" shaped block: $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

The issue here is we do not know the masses of the two pieces. However, we know both pieces have the same density. Therefore, we can set those two densities equal to one another:

$$\rho_1 = \rho_2 \Rightarrow \frac{m_1}{\forall_1} = \frac{m_2}{\forall_2} \Rightarrow \frac{m_1}{A_1 \times t} = \frac{m_2}{A_2 \times t} \Rightarrow \frac{m_1}{A_1} = \frac{m_2}{A_2} \Rightarrow m_1 = \frac{A_1}{A_2} m_2$$ ♥

This gives us a relationship between mass 1 and mass 2, which we can substitute back into the equation for center of mass of a system of particles.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{\frac{A_1}{A_2} m_2 x_1 + m_2 x_2}{\frac{A_1}{A_2} m_2 + m_2} = \frac{\frac{A_1}{A_2} x_1 + x_2}{\frac{A_1}{A_2} + 1} = \left(\frac{A_2}{A_2}\right) \frac{\frac{A_1}{A_2} x_1 + x_2}{\frac{A_1}{A_2} + 1} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

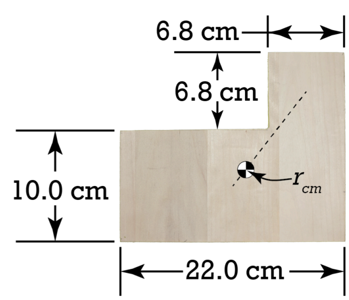
Notice in this step we have multiplied by $\frac{m_2}{m_2}$ and $\frac{m_2}{m_2} = 1$.

It turns out, if all the pieces of the object have the same density and thickness, we can substitute in the area of each piece for the mass of each piece. This can be very helpful to remember!

Now we can substitute in equations for area and numbers:

$$x_{cm} = \frac{L_1 w_1 x_1 + L_2 w_2 x_2}{L_1 w_1 + L_2 w_2} = \frac{(22)(10)(11) + (6.8)^2 (18.6)}{(22)(10) + (6.8)^2} = 12.31995 \approx 12cm$$

$$y_{cm} = \frac{L_1 w_1 y_1 + L_2 w_2 y_2}{L_1 w_1 + L_2 w_2} = \frac{(22)(10)(5) + (6.8)^2 (13.4)}{(22)(10) + (6.8)^2} = 6.45889 \approx 6.5cm$$



$$\boxed{r_{cm} \approx (12, 6.5)cm}$$

This is logical because the center of mass of the "L" shaped block should be somewhere between the centers of mass of the two pieces, but much closer to the center of piece 1.

♥ I always add a horizontal slash through my $\forall$ for volume. This is to differentiate it from v for velocity.