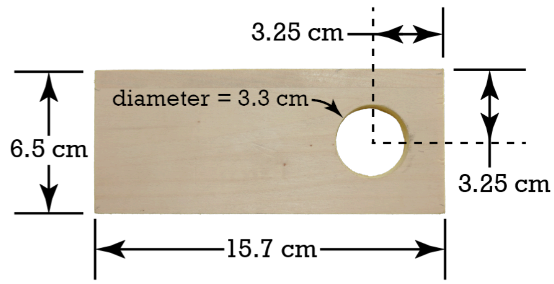
Flipping Physics Lecture Notes:

Center of Mass of an Object with a Hole

Where is the center of mass of a rectangular, constant density, constant thickness block that has a hole in it? Dimensions are shown in the illustration.



Set the zero, zero location or origin at the lower leftmost corner of the block. Split the block into symmetrical shapes with known centers of mass locations.

Piece 1 is a whole rectangular block, which is 15.7 by 6.5 cm.

- This theoretical piece does not have a hole in it.

Piece 2 is a hole with negative mass, which has a diameter of 3.3 cm.

- This theoretical piece is the hole that we need to remove from the rectangle by subtracting it.

Realize because both pieces have the same y-position center of mass, then the center of mass of the block will have the same center of mass: 3.25 centimeters up from zero.

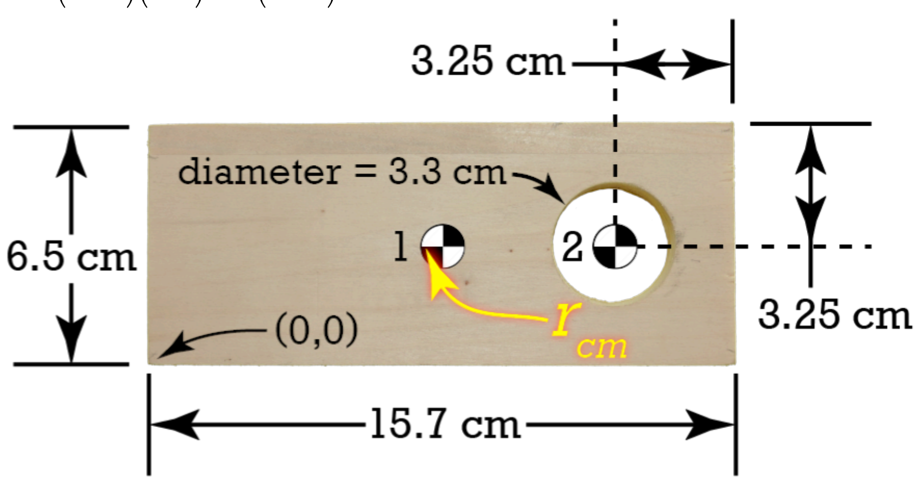
Both pieces have, due to symmetry and constant density, a center of mass at their geometric center:

$$x_1 = \frac{15.7}{2} = 7.85cm \ \& \ x_2 = 15.7 - 3.25 = 12.45cm \qquad \& \qquad r_2 = \frac{d_2}{2} = \frac{3.3}{2} = 1.65cm$$

We can use the equation for center of mass of a system of particles to determine the center of mass of the block with a hole in it: $\Rightarrow x_{cm} = \dfrac{m_1x_1 + \left(-m_2\right)x_2}{m_1 + \left(-m_2\right)} = \dfrac{A_1x_1 + \left(-A_2\right)x_2}{A_1 + -\left(A_2\right)} = \dfrac{L_1w_1x_1 - \pi\left(r_2\right)^2 x_2}{L_1w_1 - \pi\left(r_2\right)^2}$

Remember the mass of the hole is negative, which is why we are subtracting mass 2 in the equation. Also, in our previous lesson we derived why we can replace area with mass in this equation. Please see that video for that derivation. http://www.flippingphysics.com/center-of-mass-hole.html

$$\Rightarrow x_{cm} = \frac{\left(15.7\right)\left(6.5\right)\left(7.85\right) - \pi\left(1.65\right)^2\left(12.45\right)}{\left(15.7\right)\left(6.5\right) - \pi\left(1.65\right)^2} = 7.4292 \Rightarrow \boxed{x_{cm} \approx 7.4cm \ \& \ y_{cm} \approx 3.2cm}$$



This is logical because the center of mass of the block with a missing piece should be closer to the center of mass of piece 1 rather than the center of mass of the hole.