Flipping Physics Lecture Notes:

Introductory Moment of Inertia Example Problem

Example: Three 20.0-gram masses are 9.4 cm from an axis of rotation and rotating at 152 revolutions per minute. What is the moment of inertia of the three-object system? The strings holding the masses are of negligible mass.

$$I = \sum_i m_i \left(r_i\right)^2 \Rightarrow I = m_1\left(r_1\right)^2 + m_2\left(r_2\right)^2 + m_3\left(r_3\right)^2 = mr^2 + mr^2 + mr^2 = 3mr^2$$

All the masses and distances from the axis of rotation are the same:

$$m_1 = m_2 = m_3 = m \;\&\; r_1 = r_2 = r_3 = r$$

$$m = 20g \times \frac{1kg}{1000g} = 0.020kg \;\&\; r = 9.4cm \times \frac{1m}{100cm} = 0.094m$$

$$I = 3\left(0.020\right)\left(0.094\right)^2 = 0.00053016 \approx \boxed{5.3\times10^{-4} kg \bullet m^2}$$

Notice the moment of inertia of the system is independent of angular velocity.
This is the same as the mass of an object being independent of its velocity.

We can also determine the rotational kinetic energy of the system:

$$\omega = 152\frac{rev}{\min} \times \frac{1\min}{60s} \times \frac{2\pi rad}{1rev} = 15.9174\frac{rad}{s}$$

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(0.00053016\right)\left(15.9174\right)^2 = 0.0671617 \approx 6.7\times10^{-2} J \times \frac{1000mJ}{1J} = \boxed{67mJ}$$

$$\frac{kg \bullet m^2 \bullet rad^2}{s^2} = \frac{kg \bullet m^2}{s^2} = \left(\frac{kg \bullet m}{s^2}\right)m = N \bullet m = J$$