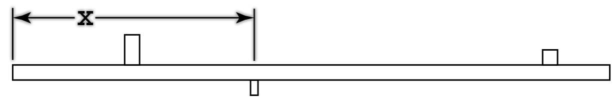
# Placing the Fulcrum on a Seesaw

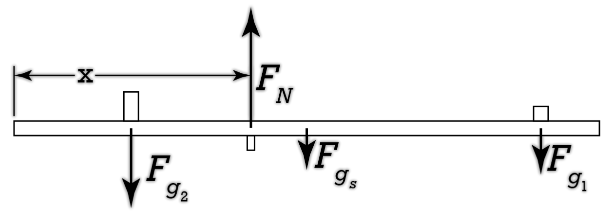
Example: A 200.0 g mass is placed at the 20.0 cm mark on a uniform 93 g meterstick. A 100.0 g mass is placed at the 90.0 cm mark. Where on the meterstick should the fulcrum be placed to balance the system?




$$m_2 = 200.0g \ @ \ 20.0cm;\ m_s = 93g;\ m_1 = 100.0g \ @ \ 90.0cm;\ x = ?$$

The system is at rest, so it is in both translational and rotational equilibrium. Therefore, the net force equals zero and the net torque about any axis of rotation equals zero. This special case is called *static* equilibrium.

Sum the forces in the y-direction on the meterstick.



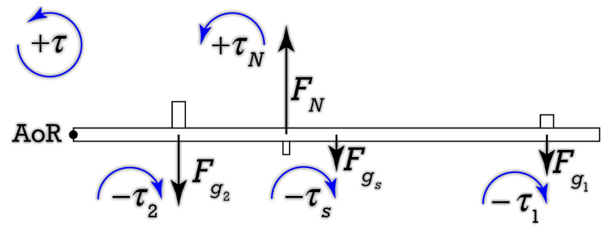

$$\sum F_y = F_N - F_{g_2} - F_{g_s} - F_{g_1} = ma_y = m(0) = 0$$

$$\Rightarrow F_N = F_{g_2} + F_{g_s} + F_{g_1} = m_2 g + m_s g + m_1 g$$

$$\Rightarrow F_N = g\left(m_2 + m_s + m_1\right)$$

Now we sum the torques on the meterstick with the axis of rotation at the left end. Assume counterclockwise, or out of the page, is positive.

$$\sum \tau_{\substack{meterstick \\ AoR\,@\,left\,end}} = -\tau_2 + \tau_N - \tau_s - \tau_1 = I\alpha = I(0) = 0$$




Torque directions:

- The force normal would cause the meterstick to rotate counterclockwise or out of the page, so the torque caused by force normal is positive.
- The force of gravity 2, force of gravity of the stick and force of gravity 1 would each cause the meterstick to rotate clockwise or into the page, so these three torques are each negative.

$$\Rightarrow -r_2 F_{g_2} \sin\theta_2 + r_N F_N \sin\theta_N - r_s F_{g_s} \sin\theta_s - r_1 F_{g_1} \sin\theta = 0$$

$$\Rightarrow -r_2 m_2 g \sin\theta_2 + r_N F_N \sin\theta_N - r_s m_s g \sin\theta_s - r_1 m_1 g \sin\theta = 0$$

$$\theta_2 = \theta_N = \theta_s = \theta_1 = 90^\circ \ \& \sin(90^\circ) = 1$$

$$\Rightarrow -r_2 m_2 g + r_N F_N - r_s m_s g - r_1 m_1 g = 0$$

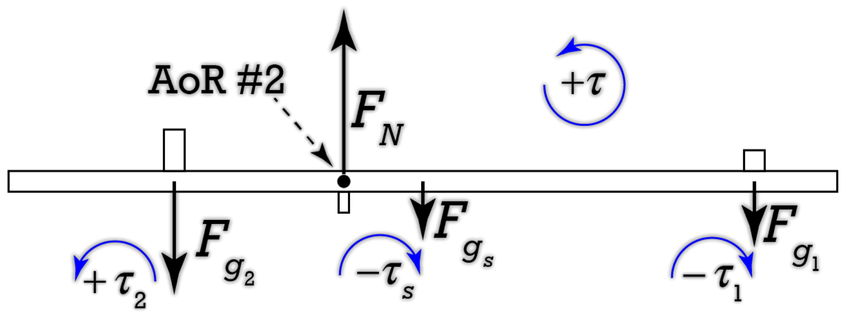
$$\Rightarrow r_N F_N = r_2 m_2 g + r_s m_s g + r_1 m_1 g = g\left(r_2 m_2 + r_s m_s + r_1 m_1\right) \Rightarrow r_N = \frac{g\left(r_2 m_2 + r_s m_s + r_1 m_1\right)}{F_N}$$

$$\Rightarrow r_N = \frac{g\left(r_2 m_2 + r_s m_s + r_1 m_1\right)}{g\left(m_2 + m_s + m_1\right)} = \frac{r_2 m_2 + r_s m_s + r_1 m_1}{m_2 + m_s + m_1} = \frac{(20)(200) + (50)(93) + (90)(100)}{200 + 93 + 100}$$

$$\Rightarrow r_N = 44.9109 \approx \boxed{45\text{cm}}$$

Alternate solution without using Newton's Second Law:

Now we sum the torques on the meterstick with the axis of rotation at the Force Normal. Assume counterclockwise or out of the page is positive.




$$\sum \tau_{\substack{meterstick \\ AoR\,\#2\,@\,fulcrum}} = \tau_2 + \cancel{\tau_N} - \tau_s - \tau_1 = I\alpha = I(0) = 0$$

Torque directions:

- Force of gravity 2 would cause the meterstick to rotate counterclockwise or out of the page, so the torque caused by force of gravity 2 is positive.
- The force normal acts right at the axis of rotation, therefore, the "r" value for the force normal is zero, and the torque caused by the force normal is zero and has no direction.
- The force of gravity of the stick and force of gravity 2 would both cause the meterstick to rotate clockwise or into the page, so the torques caused by force of gravity of the stick and force of gravity #2 are both negative.
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$$\Rightarrow r_2 F_{g_2} \sin\theta_2 - r_s F_{g_s} \sin\theta_s - r_1 F_{g_1} \sin\theta_1 = 0$$

$$\theta_2 = \theta_s = \theta_1 = 90^\circ \ \& \sin(90^\circ) = 1$$

$$\Rightarrow r_2 m_2 \cancel{g} - r_s m_s \cancel{g} - r_1 m_1 \cancel{g} = 0 \Rightarrow r_2 m_2 - r_s m_s - r_1 m_1 = 0$$

(everybody brought g, the acceleration to gravity, to the party!)

Define x as the distance from the left end of the meterstick to the axis of rotation.

$$x = 20 + r_2 \Rightarrow r_2 = x - 20 \ \& \ 50 = x + r_s \Rightarrow r_s = 50 - x \ \& \ 90 = x + r_1 \Rightarrow r_1 = 90 - x$$

$$r_2 m_2 - r_s m_s - r_1 m_1 = 0 \Rightarrow (x-20)(200) - (50-x)(93) - (90-x)(100) = 0$$

$$\Rightarrow 200x - 4000 - 4650 + 93x - 9000 + 100x = 0$$

$$\Rightarrow 200x + 93x + 100x - 4000 - 4650 - 9000 = 0$$

$$\Rightarrow (200 + 93 + 100)x - (17650) = 0 \Rightarrow 393x = 17650$$

$$\Rightarrow x = \frac{17650 g \cdot cm}{393 g} = 44.911 cm \approx \boxed{45 cm}$$