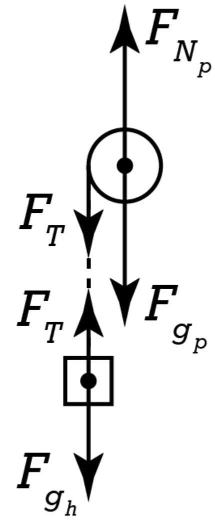
Flipping Physics Lecture Notes:

Graphing the Rotational Inertia of an Irregular Shape

We have discussed the equations for the rotational inertia of common shapes. See: Moments of Inertia of Rigid Objects with Shape - https://www.flippingphysics.com/moment-of-inertia-rigid-objects.html However, we should also know how to measure the rotational inertia of irregular shapes. The Rotational Inertia Demonstrator from Arbor Scientific is a pulley; however, it does not fit any of the standard shapes. Our goal is to create a graph where the slope of the best-fit line is the rotational inertia of the rotational inertia demonstrator. Let's start with free body diagrams.



We can sum the torques on the pulley with an axis of rotation and the center of the pulley and define counterclockwise, or out of the board, as positive. Because they both act on the axis of rotation, the force normal and force of gravity which act on the pulley cause zero torque on the pulley. The torque caused by the force of tension on the pulley causes the pulley to rotate in the positive direction.

$$\sum \vec{\tau}_{\substack{pulley \\ AoR\ @\ pulley\ center}} = \vec{\tau}_{F_T} = I\vec{\alpha} \Rightarrow rF_T \sin\theta = I\alpha \Rightarrow RF_T \sin(90) = I\alpha \Rightarrow RF_T = I\alpha$$

Compare this equation to the slope intercept form of a line equation:

$$y = (slope)x + b \Rightarrow y = RF_T;\ slope = I;\ x = \alpha;\ b = 0$$

The pulley has three different radii: $R_1 = 0.0202m;\ R_2 = 0.0286m;\ R_3 = 0.0385m$

The force of tension can be measured using a force sensor as a part of the hanging mass. Because the string has the same force of tension on both ends, the force of tension we measure on the hanging mass is the same as the force of tension which acts on the pulley.

The angular acceleration of the pulley needs to be measured using a uniformly angularly accelerated motion equation: $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2 = \frac{1}{2}\alpha\Delta t^2 \Rightarrow \alpha = \frac{2\Delta\theta}{\Delta t^2}$

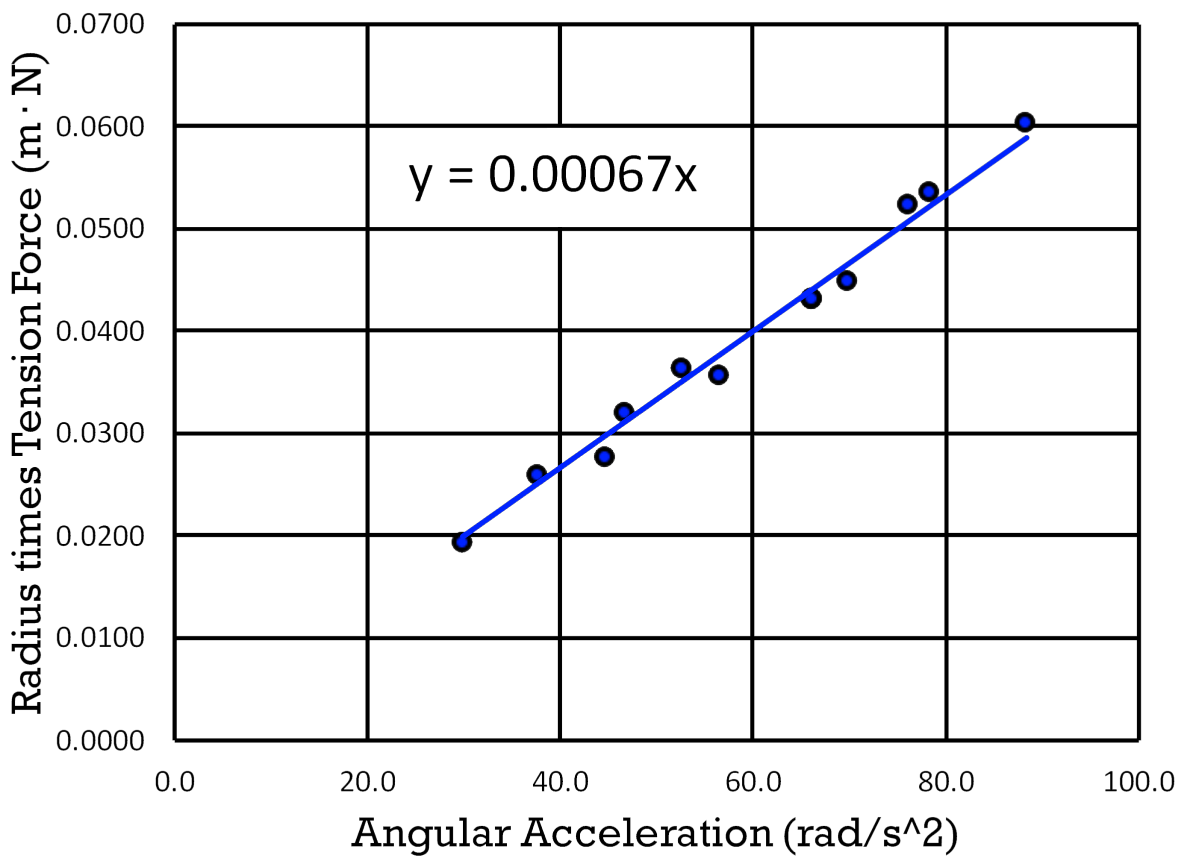
Therefore we need change in time, angular displacement, and zero initial angular velocity.

Here is a sample calculation for the angular acceleration of the first trial:

$$\Delta t = 55frames \times \frac{1\sec}{60frames} = 0.91\bar{6}\sec;\ \Delta\theta = 2\text{rev} \times \frac{2\pi rad}{1rev} = 4\pi rad;\ \omega_i = 0$$

$$\alpha = \frac{2\Delta\theta}{\Delta t^2} = \frac{(2)(4\pi)}{(0.91\bar{6})^2} = 29.910\frac{rad}{s^2}$$

Trial	Hanging Mass (kg)	Pulley Radius (m)	Force of Tension (N)	α (rad/s^2)	R * F_T (m·N)
1	0.103	0.0202	0.955	29.9	0.0193
2	0.153	0.0202	1.365	44.7	0.0276
3	0.203	0.0202	1.762	56.5	0.0356
4	0.253	0.0202	2.131	66.1	0.0431
5	0.103	0.0286	0.904	37.7	0.0258
6	0.153	0.0286	1.268	52.5	0.0363
7	0.203	0.0286	1.566	69.8	0.0448
8	0.253	0.0286	1.871	78.3	0.0535
9	0.103	0.0385	0.829	46.7	0.0319
10	0.153	0.0385	1.122	66.1	0.0432
11	0.203	0.0385	1.361	76.0	0.0524
12	0.253	0.0385	1.566	88.4	0.0603



The best fit line equation y = 0.00067x means $RF_T = 0.00067\alpha$ which means, because $RF_T = I\alpha$, the rotational inertia of the Rotational Inertia Demonstrator is 0.00067 kg·m^2.

Confirming the units:

$$RF_T = I\alpha \Rightarrow I = \frac{RF_T}{\alpha} \Rightarrow \frac{m \cdot N}{\frac{rad}{s^2}} = \left(m \cdot \frac{kg \cdot m}{s^2}\right)\left(\frac{s^2}{rad}\right) = kg \cdot m^2$$