Flipping Physics Lecture Notes:

Rolling Without Slipping
Introduction and Demonstrations

Realize these lecture notes will be much better understood with the visuals in the video at https://www.flippingphysics.com/rolling-without-slipping.html

An object which is rolling without slipping stays in contact with the ground and does not slide relative to the ground. Rolling without slipping combines translational with rotational motion.

Every part of the object in translational motion moves with the same velocity. We will define that as the velocity of the center of mass of the translational object: $v_T = v_{cm}$

The center of mass of the rotational object has zero velocity: $v_{R\,cm} = 0$

The outer edge of the rotational object has a velocity equal to its tangential velocity: $v_{R\,edge} = R\omega$

The velocity of every point on the object rolling without slipping equals the addition of the tangential velocity and the rotational velocity. Because the point of contact of the object rolling without slipping does not move relative to the stationary ground, the velocity at the point of contact is zero. Therefore, the velocity of the center of mass of an object rolling without slipping equals the radius of the object times the angular velocity of the object:

$$\vec{v}_{bottom} = \vec{v}_T + \vec{v}_{R\,edge} \Rightarrow v_{bottom} = v_{cm} - R\omega = 0 \Rightarrow v_{cm} = R\omega$$

This is very much like the tangential velocity equation, however, "r" is the radius of the object: $v_t = r\omega$

The kinetic energy of an object rolling without slipping includes translational and rotational kinetic energies: $KE_{total} = KE_T + KE_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Notice the equations for distance travelled by and acceleration of an object rolling without slipping can also be derived this same way and are also similar to their arc length and tangential acceleration counterparts: $x_{cm} = R\Delta\theta$ & $s = r\Delta\theta$; $v_t = r\omega$ & $v_{cm} = R\omega$; $a_{cm} = R\alpha$ & $a_t = r\alpha$