Flipping Physics Lecture Notes:
Introduction to Tip-to-Tail Vector Addition, Vectors and Scalars

Slow Velocity Racer: Distance = 1.00 m = 1.00 x 10^3 mm & Time = 23.8 seconds

$$speed = \frac{distance}{time} = \frac{1000mm}{23.8\sec} = 42.0168 \approx 42\frac{mm}{s}$$

$$speed_{bottom\,track} = 42\frac{mm}{s} + 42\frac{mm}{s} = 84\frac{mm}{s}$$

$$speed_{in\,the\,middle} = 42\frac{mm}{s} + 0 = 42\frac{mm}{s}$$

$$speed_{top\,track} = 42\frac{mm}{s} - 42\frac{mm}{s} = 0$$

This is Tip-to-Tail (or Tail to Tip) Vector Addition:

Vector: A quantity that has both magnitude and direction.
Vector examples: Displacement, Velocity, Acceleration, Force, Torque, and Momentum

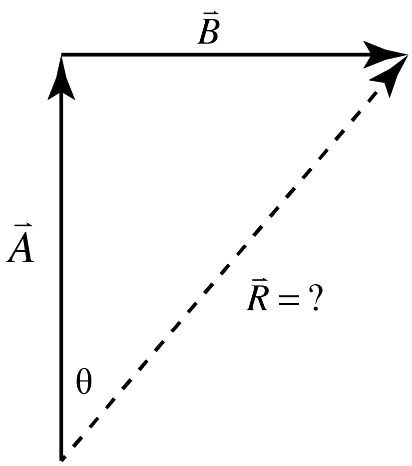
Scalar: A quantity that has magnitude only (no direction).
Scalar examples: Distance, Speed, Time, Volume, Density, and Money

Magnitude: The numerical amount of the quantity.

Flipping Physics Lecture Notes:
Introductory Tip-to-Tail Vector Addition Problem

Determining the velocity of the track: $v=\frac{\Delta x}{\Delta t}=\frac{600mm\,North}{12.2\text{ sec}}=49.180\approx 49\frac{mm}{s}North$

The Velocity Vectors: $\left[Track\right]\vec{A}=49\frac{mm}{s}North$ & $\left[racecar\right]\vec{B}=42\frac{mm}{s}East$



$\vec{A}+\vec{B}=\vec{R}=?$

This is called Tip-to-Tail Vector Addition. To add vectors $\vec{A}$ and $\vec{B}$, the tip of vector $\vec{A}$ is placed on the tail of vector $\vec{B}$. The result is called the Resultant Vector $\vec{R}$, which is what we are trying to find.

We can find the magnitude of $\vec{R}$ by using the Pythagorean theorem:

$$a^2+b^2=c^2\Rightarrow R^2=A^2+B^2\Rightarrow R=\sqrt{A^2+B^2}=\sqrt{49^2+42^2}=64.537\approx 65\frac{mm}{s}$$

We can find the direction of $\vec{R}$ by using SOH CAH TOA:

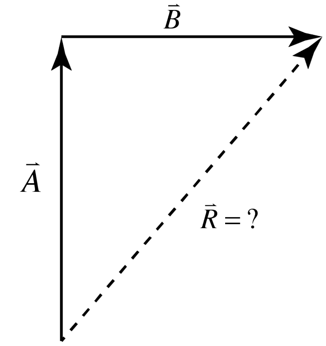
$$\tan\theta=\frac{O}{A}=\frac{B}{A}=\frac{42}{49}\Rightarrow\theta=\tan^{-1}\left(\frac{42}{49}\right)=40.601^\circ\approx 41^\circ$$

Therefore the resultant velocity *vector* $\vec{R}$ is: $\boxed{\vec{R}\approx 65\frac{mm}{s}@41^\circ\ E\ of\ N}$

In other words: $\vec{A}+\vec{B}=\vec{R}\Rightarrow 49\frac{mm}{s}North+42\frac{mm}{s}East\approx 65\frac{mm}{s}@41^\circ E\,of\,N$

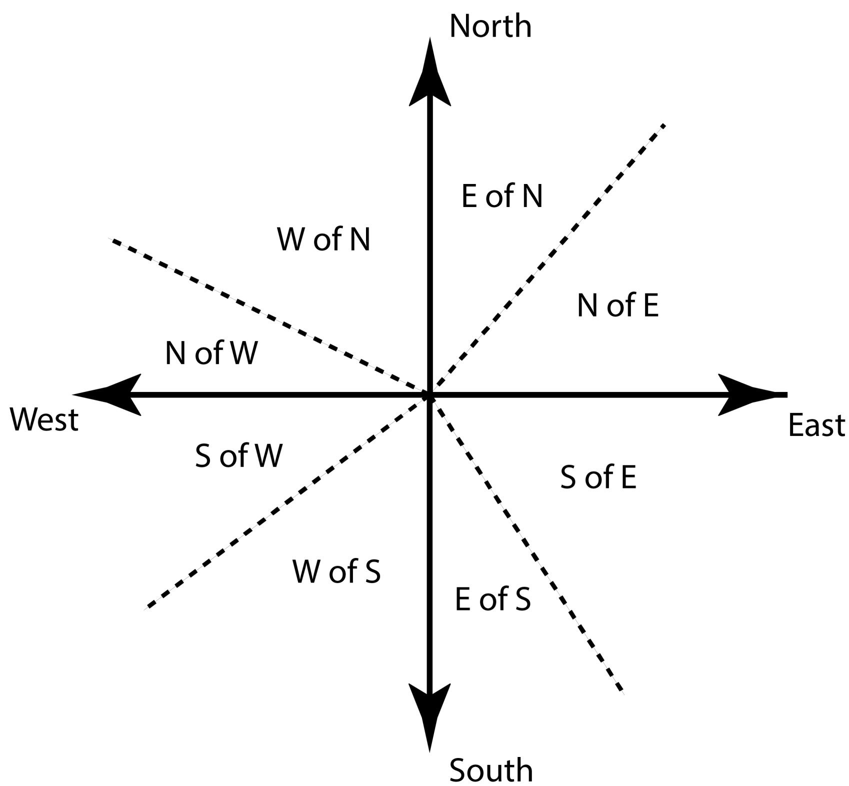
(The cardinal direction East of North will be explained in the next video)

Flipping Physics Lecture Notes:
How to use Cardinal Directions with Vectors



In our previous lesson we added the two vectors $\vec{A}$ and $\vec{B}$, using tip-to-tail vector addition to get a resultant vector $\vec{R} \approx 65\frac{mm}{s} @ 41° \; E \; of \; N$. Now we are going to understand what 41° East of North means.

East of North is better described as East "from" North, because it means that the angle is measured Eastward from the Northward direction. This means that the eight ways to describe the direction of an angle are can be illustrated like this:

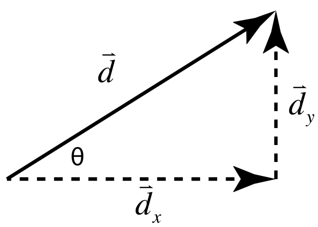


This means that our original vector $\vec{R} \approx 65\frac{mm}{s} @ 41° \; E \; of \; N$ is equivalent to $\vec{R} \approx 65\frac{mm}{s} @ 49° \; N \; of \; E$ because 41° + 49° = 90°.

Also the term Northeast (or NE) means exactly halfway between North and East or 45° N of E or 45° E of N. This is also true of Southeast (SE), Southwest (SW) and Northwest (NW).

Flipping Physics Lecture Notes:
Introduction to Vector Components

Starting with the displacement vector for our Slow Velocity Racer, $\vec{d} = 90.0cm\ @\ 32°\ N\ of\ E$, we can determine the components, or pieces, of displacement $\vec{d}$ in the x and y directions.



We can use SOH to find the displacement in the y direction:

$$\sin\theta = \frac{O}{H} = \frac{\vec{d}_y}{\vec{d}} \Rightarrow \vec{d}_y = \vec{d}\sin\theta = 90\sin 32 = 47.693 \approx 48cm$$

And we can use CAH to find the displacement in the x direction:

$$\cos\theta = \frac{A}{H} = \frac{\vec{d}_x}{\vec{d}} \Rightarrow \vec{d}_x = \vec{d}\cos\theta = 90\cos(32) = 76.324 \approx 76cm$$

So have broken our displacement vector $\vec{d}$ in to its components in the x and y direction:

$$\boxed{\vec{d}_y \approx 48cm\ \&\ \vec{d}_x \approx 76cm}$$

You can also say “resolve” vectors in to components. I prefer “break” vectors in to components, it has that hard “k” sound, which makes it more fun to say.

$\vec{d}_y \approx 48cm\ \&\ \vec{d}_x$ are the components of $\vec{d}$ because they add up to the vector $\vec{d}$. [$\vec{d}_y + \vec{d}_x = \vec{d}$] We can show this by working this problem now in reverse. First we find the magnitude of $\vec{d}$ by using the Pythagorean theorem.

$$a^2 + b^2 = c^2 \Rightarrow d^2 = d_x^2 + d_y^2 \Rightarrow d = \sqrt{d_x^2 + d_y^2} = \sqrt{(76.324)^2 + (47.493)^2} = 89.894cm$$

And then we can find the direction by using TOA:

$$\tan\theta = \frac{O}{A} = \frac{\vec{d}_y}{\vec{d}_x} \Rightarrow \theta = \tan^{-1}\left(\frac{\vec{d}_y}{\vec{d}_x}\right) = \tan^{-1}\left(\frac{47.493}{76.324}\right) = 31.892°$$

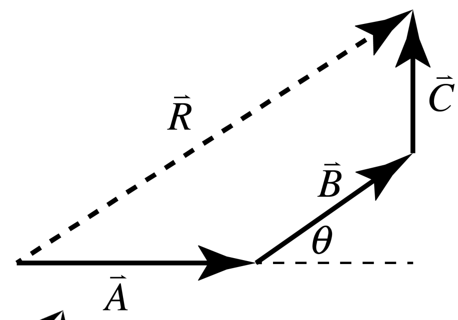
Therefore, rounded to 2 sig figs, we get:

$$\vec{d} \approx 9.0x10^1 cm\ @\ 32°\ N\ of\ E$$

Which is the displacement vector we started with.

Also notice that $\vec{d}_y \approx 48cm\ \&\ \vec{d}_x \approx 76cm$ are vectors because they do have both magnitude and direction. The subscripts of y & x illustrate the direction and both numbers are positive. This means that $\vec{d}_y \approx 48cm$ is 48 cm in the positive y direction and $\vec{d}_x \approx 76cm$ is 76 cm in the positive x direction.

Flipping Physics Lecture Notes:
Introductory Vector Addition Problem using Component Vectors



Example Problem: Slow Velocity Racer races 50.0 cm East, then turns 35° North of East and scoots for 40.0 cm. She then turns and moseys another 30.0 cm North. What was her total displacement?

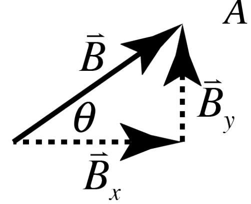
$\vec{A} = 50.0cm\ E$, $\vec{B} = 40.0cm\ N\ of\ E$, $\vec{C} = 30.0cm\ N$ &

$\vec{A} + \vec{B} + \vec{C} = \vec{R} = ?$

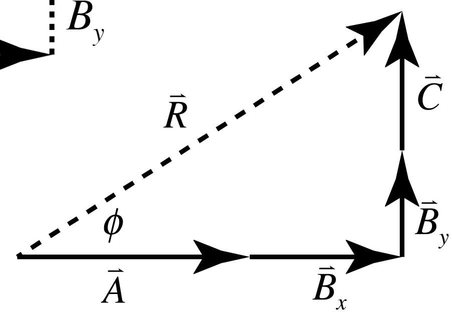
Break vector $\vec{B}$ in to its components.

$$\sin\theta = \frac{O}{H} = \frac{\vec{B}_y}{\vec{B}} \Rightarrow \vec{B}_y = \vec{B}\sin\theta = 40\sin(35) = 22.943cm$$

$$\cos\theta = \frac{A}{H} = \frac{\vec{B}_x}{\vec{B}} \Rightarrow \vec{B}_x = \vec{B}\cos\theta = 40\cos(35) = 32.766cm$$



Redraw the Vector Diagram.



And now we have a right triangle and can use SOH CAH TOA and the Pythagorean theorem.

$$a^2 + b^2 = c^2 \Rightarrow R^2 = (A + B_x)^2 + (B_y + C)^2$$

$$\Rightarrow R = \sqrt{(A + B_x)^2 + (B_y + C)^2} = \sqrt{(50 + 32.766)^2 + (22.943 + 30)^2} = 98.251cm$$

$$\tan\phi = \frac{O}{A} = \frac{B_y + C}{A + B_x} \Rightarrow \phi = \tan^{-1}\left(\frac{B_y + C}{A + B_x}\right) = \tan^{-1}\left(\frac{22.943 + 30}{50 + 32.766}\right) = 32.606°$$

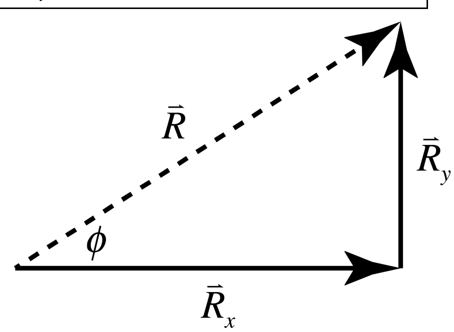
$$\Rightarrow \boxed{\vec{R} \approx 98cm\ @\ 33°\ N\ of\ E}$$

Flipping Physics Lecture Notes:
Using a Data Table to Make Vector Addition Problems Easier

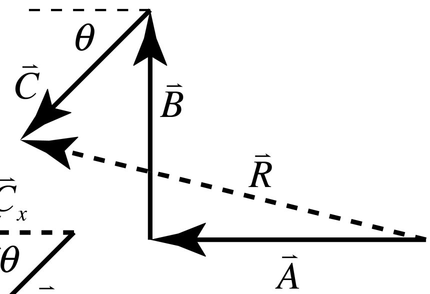
An Easy way to see that this works is by using a table.

Vector	x-direction (cm)	y-direction (cm)
$\vec{A}$	50	0
$\vec{B}$	32.766	22.943
$\vec{C}$	0	30
$\vec{R}$	$\vec{R}_x = 50 + 32.766 + 0 = 82.766$	$\vec{R}_y = 0 + 22.943 + 30 = 52.943$

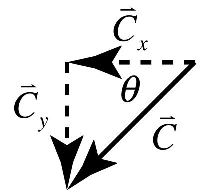
And you can see that the components $\vec{R}_x$ and $\vec{R}_y$ add up to vector $\vec{R}$.



Example Problem: Slow Velocity Racer races 60.0 cm West, then turns North and drives for 50.0 cm. She then turns and slowly meanders another 40.0 cm SW. What was her total displacement?



$\vec{A} = 60.0cm\ W$, $\vec{B} = 50.0cm\ N$, $\vec{C} = 40.0cm\ SW$ &
$\vec{A} + \vec{B} + \vec{C} = \vec{R} = ?$



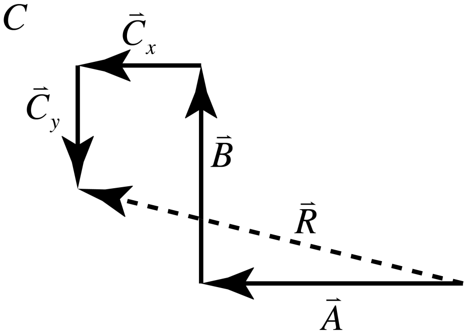
Break vector $\vec{C}$ in to its components.

$$\sin\theta = \frac{O}{H} = \frac{\vec{C}_y}{C} \Rightarrow \vec{C}_y = \vec{C}\sin\theta = 40\sin(45)$$

$$\Rightarrow \vec{C}_y = 28.284cm\ South = -28.284cm$$

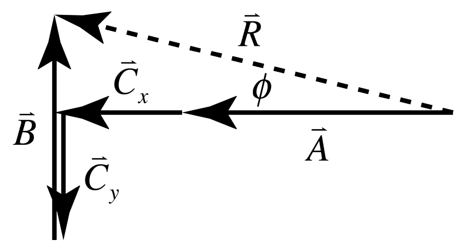
$$\cos\theta = \frac{A}{H} = \frac{\vec{C}_x}{\vec{C}} \Rightarrow \vec{C}_x = \vec{C}\cos\theta = 40\cos(45)$$

$$\Rightarrow \vec{C}_x = 28.284cm\ West = -28.284cm$$



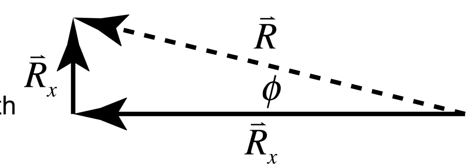
You should recognize that the sine and cosine of 45° are exactlty the same and therefore the magnitudes of $\vec{C}_x$ & $\vec{C}_y$ are the same (but the directions are different).

Redraw the Vector Diagram. Some of you might not see the right triangle, however, remember the Associative Property of Vector Addition! The order you add the vectors in doesn't matter. So let's try adding them in a different order …



So now we have a right triangle and set up a table to help determine the components of the resultant vector:

Vector	x-direction (cm)	y-direction (cm)
$\vec{A}$	-60	0
$\vec{B}$	0	50
$\vec{C}$	-28.284	-28.284
$\vec{R}$	$\vec{R}_x = -60 - 28.284 = -88.284$	$\vec{R}_y = 50 - 28.284 = 21.716$



And if you didn't see the right triangle before, perhaps you can see it now with the redrawn vector diagram with the components of the Resultant Vector.

Now we can use SOH CAH TOA and the Pythagorean theorem to find the Resultant Vector, $\vec{R}$.

$$a^2 + b^2 = c^2 \Rightarrow R^2 = R_x^2 + R_y^2 \Rightarrow R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-88.284)^2 + (21.716)^2} = 90.916cm$$

$$\&\ \sin\phi = \frac{O}{H} = \frac{R_y}{R} \Rightarrow \phi = \sin^{-1}\left(\frac{R_y}{R}\right) = \sin^{-1}\left(\frac{21.716}{90.916}\right) = 13.819°$$

$$\Rightarrow \boxed{\vec{R} \approx 90.9cm\ @\ 13.8°\ N\ of\ W}$$