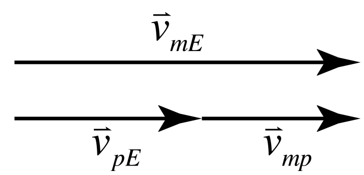
Flipping Physics Lecture Notes:
Introduction to Relative Motion using a Quadcopter Drone

Up to this point all velocities have been understood to be relative to the ground or the Earth. And, unless otherwise noted, that's the way it will continue. However, today we are going to look at velocities relative to objects that are not the Earth. Let's start with the velocities of two cars relative to the Earth:

$\vec{v}_{mE} = 24\frac{mi}{hr}E$ (Read: The velocity of the minivan with respect to the Earth is 24 miles per hour East)

$\vec{v}_{pE} = 13\frac{mi}{hr}E$ (Read: The velocity of the prius with respect to the Earth is 13 miles per hour East)

Now let's find the velocity of the minivan with respect to the prius: $\vec{v}_{mp} = ?$ In other words, while I am driving the prius and I look out the window to watch the minivan go by, at what velocity would I measure the minivan to be moving? Some of you may be able to immediately identify that the answer is 11 miles per hour East, however, in order to understand more complicated relative motion problems, let's walk through the math. The vectors look like this:



From the vector diagram you can see that $\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp}$ which we can solve for $\vec{v}_{mp}$:

$$\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp} \Rightarrow \vec{v}_{mE} - \vec{v}_{pE} = \vec{v}_{mp} \Rightarrow \vec{v}_{mp} = \vec{v}_{mE} - \vec{v}_{pE} = 24\frac{mi}{hr}E - 13\frac{mi}{hr}E = \boxed{11\frac{mi}{hr}E}$$

You should remember that taking the negative of a vector changes the direction by 180°. In terms of relative motion, it switches the order of the subscripts. In other words: $-\vec{v}_{pE} = \vec{v}_{Ep} = -13\frac{mi}{hr}E = 13\frac{mi}{hr}W$ Which is useful because it means we can substitute $\vec{v}_{Ep}$ into the equation: $\vec{v}_{mp} = \vec{v}_{mE} - \vec{v}_{pE} \Rightarrow \vec{v}_{mp} = \vec{v}_{mE} + \vec{v}_{Ep}$ and notice what happens to the subscripts on the right hand side of the equation; when you add two vectors together like this, the Earth drops out.

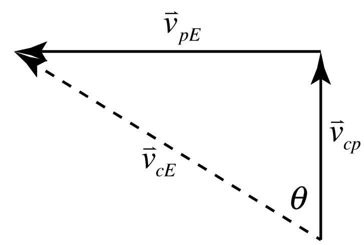
Read that last equation very carefully, $\vec{v}_{mp} = \vec{v}_{mE} + \vec{v}_{Ep}$: The velocity of the minivan with respect to the prius is the same as the velocity of the minivan with respect to the Earth plus the velocity of the Earth with respect to the prius. Understanding that the Earth drops out of the equation will make more complicated relative motion problems easier.

The same is actually true of the first equation: $\vec{v}_{mE} = \vec{v}_{pE} + \vec{v}_{mp}$, The velocity of the prius drops out of the right hand side of the equation to give us the velocity of the minivan with respect to the Earth.

Now let's find the velocity of the prius with respect to the minivan, $\vec{v}_{pm} = ?$ In other words, while my wife drives the minivan and she looks out the window to watch the prius as she passes it, at what velocity would she measure the prius to be moving? The solution is actually rather simple: $\vec{v}_{pm} = -\vec{v}_{mp} = -11\frac{mi}{hr}E = \boxed{11\frac{mi}{hr}W}$

Flipping Physics Lecture Notes:
An Introductory Relative Motion Problem

Example Problem: A toy car travels at 46 mm/s N relative to a piece of paper that is moving at 75 mm/s W relative to the Earth. (a) What is the velocity of the toy car relative to the Earth? (b) If the width of the paper necessary for the toy car to cross is 69.2 cm, how far did the toy car actually travel? (c) How long did it take the toy car to cross the paper?



Givens: $\vec{v}_{cp} = 46\frac{mm}{s}N$, $\vec{v}_{pE} = 75\frac{mm}{s}W$, $\vec{v}_{cE} = ?$

$\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE}$ & $a^2 + b^2 = c^2 \Rightarrow v_{cp}^2 + v_{pE}^2 = v_{cE}^2$

(The velocity of the car with respect to the Earth is the same as the velocity of the car with respect to the paper plus the velocity of the paper with respect to the Earth; the paper drops out of the equation.)

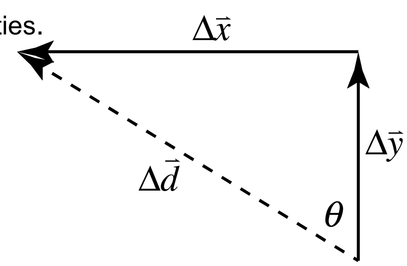
$\Rightarrow v_{cE} = \sqrt{v_{cp}^2 + v_{pE}^2} = \sqrt{46^2 + 75^2} = 87.983\frac{mm}{s} \approx 88\frac{mm}{s}$ (magnitude only)

Now we need the direction of the velocity *vector*.

$$\tan\theta = \frac{O}{A} = \frac{v_{pE}}{v_{cp}} \Rightarrow \theta = \tan^{-1}\left(\frac{v_{pE}}{v_{cp}}\right) = \tan^{-1}\left(\frac{75}{46}\right) = 58.478° \approx 58°$$

$$\Rightarrow \vec{v}_{cE} \approx \boxed{88\frac{mm}{s} @ 58°\,W\,of\,N}$$

Part (b): Now we draw a similar triangle with displacements instead of velocities.
$\Delta d = ?$ (magnitude only, not displacement)



New given for part (b): $\Delta\vec{y} = 69.2cm\ N \times \frac{1m}{100cm} \times \frac{1000mm}{1m} = 692mm\ N$

Because they are similar triangles, cosθ will have the same value for both.

$$\cos\theta = \frac{A}{H} = \frac{v_{cp}}{v_{cE}} = \frac{\Delta y}{\Delta d} \Rightarrow \frac{v_{cp}}{v_{cE}}(\Delta d) = \frac{\Delta y}{\Delta d}(\Delta d) \Rightarrow \frac{v_{cp}}{v_{cE}}(\Delta d) = \Delta y$$

$$\Rightarrow \frac{v_{cp}}{v_{cE}}(\Delta d)\left(\frac{v_{cE}}{v_{cp}}\right) = \Delta y\left(\frac{v_{cE}}{v_{cp}}\right) \Rightarrow (\Delta d) = \frac{(\Delta y)(v_{cE})}{v_{cp}} = \frac{(692)(87.983)}{46} = 1323.57mm \approx \boxed{1300mm}$$

Part (c): The car is moving at a constant velocity. $\Delta t = ?$

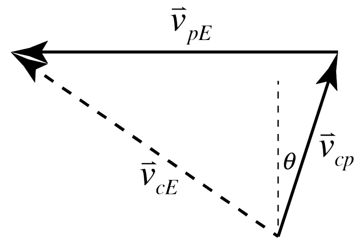
$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t} \Rightarrow v_{cE} = \frac{\Delta d}{\Delta t} \Rightarrow \Delta t = \frac{\Delta d}{v_{cE}} = \frac{1323.57mm}{87.983\frac{mm}{s}} = 15.043 \approx \boxed{15\sec}$$

Or we could have used the y-direction instead:

$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t} \Rightarrow v_{cp} = \frac{\Delta y}{\Delta t} \Rightarrow \Delta t = \frac{\Delta 7}{v_{cp}} = \frac{692mm}{46\frac{mm}{s}} = 15.043 \approx \boxed{15\sec}$$

Flipping Physics Lecture Notes:
An introductory Relative Motion Problem with Vector Components

Example Problem: A toy car travels at 42 mm/s @ 18° E of N relative to a piece of paper that is moving at 71 mm/s W relative to the Earth. What is the velocity of the toy car relative to the Earth?

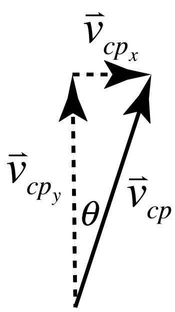


Givens: $\vec{v}_{cp} = 42\frac{mm}{s} @ 18° E\,of\,N$, $\vec{v}_{pE} = 71\frac{mm}{s}W$, $\vec{v}_{cE} = ?$

$\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE}$

(The velocity of the car with respect to the Earth is the same as the velocity of the car with respect to the paper plus the velocity of the paper with respect to the Earth; the paper drops out of the equation.)

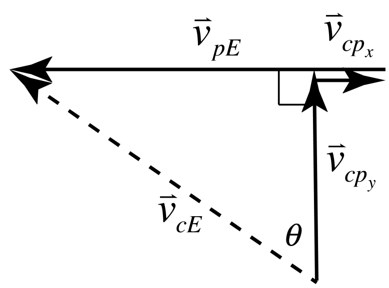
We can't use Pythagorean theorem or SOH CAH TOA because we don't have a right triangle. We need to resolve or break $\vec{v}_{cp}$ in to its components first.



$$\sin\theta = \frac{O}{H} = \frac{v_{cp_x}}{v_{cp}} \Rightarrow v_{cp_x} = v_{cp}\sin\theta = (42)\sin(18) = 12.979\frac{mm}{s}$$

$$\cos\theta = \frac{A}{H} = \frac{v_{cp_y}}{v_{cp}} \Rightarrow v_{cp_y} = v_{cp}\cos\theta = (42)\cos(18) = 39.944\frac{mm}{s}$$

Now we need to redraw the vector diagram.
And you can see that we now have a right triangle.



$$a^2 + b^2 = c^2 \Rightarrow v_{cE}^2 = \left(v_{pE} + v_{cp_x}\right)^2 + v_{cp_y}^2 \Rightarrow v_{cE} = \sqrt{\left(v_{pE} + v_{cp_x}\right)^2 + v_{cp_y}^2}$$

$$\Rightarrow v_{cE} = \sqrt{(-71 + 12.979)^2 + (39.944)^2} = 70.442 \approx 7.0 \times 10^1\frac{mm}{s}$$

That is only the magnitude, now we need the direction.

$$\cos\theta = \frac{A}{H} = \frac{v_{cp_y}}{v_{cE}} \Rightarrow \theta = \cos^{-1}\left(\frac{v_{cp_y}}{v_{cE}}\right) = \cos^{-1}\left(\frac{39.944}{70.442}\right) = 55.455 \approx 55°$$

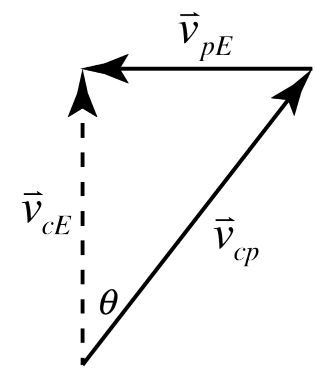
$$\Rightarrow \vec{v}_{cE} \approx \boxed{7.0 \times 10^1\frac{mm}{s} @ 55° W\,of\,N}$$

Flipping Physics Lecture Notes:
Relative Motion Problem: Solving for the angle of the moving object

Example Problem: A toy car is moving on a piece of paper. The toy car can travel at a speed of 47 mm/s relative to the paper. The paper has a velocity of 29 mm/s W relative to the Earth. (a) At what angle should the toy car travel relative to the paper such that the car will move due north relative to the Earth? (b) What is the speed of the car relative to the Earth?

Givens: $\vec{v}_{cp} = 47\frac{mm}{s}$ @ $\theta = ?$, $\vec{v}_{pE} = 29\frac{mm}{s}W$, $\vec{v}_{cE}$ is North.

Because the paper is traveling west and the velocity of the car with respect to the Earth is North, the car must be traveling in the northeasterly direction, we just need to figure out exactly what direction. The vector diagram looks like this:



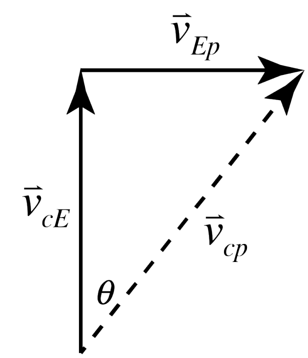
We get the same vector addition formula we have had now three times in a row: $\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE}$ However, now we are not solving for $\vec{v}_{cE}$, we are solving for the direction of $\vec{v}_{cp}$. Which means we need to rearrange the formula.

$\vec{v}_{cE} = \vec{v}_{cp} + \vec{v}_{pE} \Rightarrow \vec{v}_{cp} = \vec{v}_{cE} - \vec{v}_{pE}$

And remember $\vec{v}_{Ep} = -\vec{v}_{pE}$ so: $\vec{v}_{cp} = \vec{v}_{cE} - \vec{v}_{pE} \Rightarrow \vec{v}_{cp} = \vec{v}_{cE} + \vec{v}_{Ep}$

(The velocity of the car with respect to the paper is the same as the velocity of the car with respect to the Earth plus the velocity of the Earth with respect to the paper. Remember, the Earth drops out of the equation when it is the middle subscript.)

Remembering that switching the order of the subscripts rotates the vector 180°, we can draw a new velocity vector diagram:



Now we have a right triangle, vector addition problem.

$$\sin\theta = \frac{O}{H} = \frac{v_{Ep}}{v_{cp}} \Rightarrow \theta = \sin^{-1}\left(\frac{v_{Ep}}{v_{cp}}\right) = \sin^{-1}\left(\frac{29}{47}\right) = 38.099 \approx \boxed{38^\circ\ E\,of\,N}$$

Part (b)

$$a^2 + b^2 = c^2 \Rightarrow v_{cE}^2 + v_{Ep}^2 = v_{cp}^2 \Rightarrow v_{cE}^2 = v_{cp}^2 - v_{Ep}^2 \Rightarrow v_{cE} = \sqrt{v_{cp}^2 - v_{Ep}^2} = \sqrt{47^2 - 29^2} = 36.986\frac{mm}{s} \approx \boxed{37\frac{mm}{s}}$$

You may have noticed that the length of the vectors here and in the video don't quite match. Sadly, the velocity of the paper with respect to the Earth was actually 39 mm/s (which is what I used in the video illustrations to match the demonstration), however, at some point between filming the demonstration and writing the script and lecture notes, it got changed to 29 mm/s. I didn't notice until I was *completely* done with the video. Because the physics learning is unaffected and unlimited resources and time are decidedly *not* at my disposal, I made the difficult decision not to refilm the entire video. I hope this in no way tarnishes your respect for me. ☺