Flipping Physics Lecture Notes:

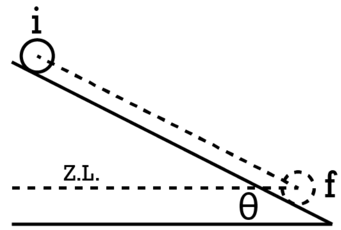
Rolling Acceleration Down an Incline

Example: Determine the acceleration of a uniform, solid cylinder rolling without slipping down an incline with incline angle θ. The rotational inertia of a uniform, solid cylinder about its long cylindrical axis is ½MR^2. Assume the cylinder starts from rest.

$$a = ?;\ \text{Uniform Solid Cylinder; Rolling without Slipping; Incline Angle} = \theta;\ I = \frac{1}{2}MR^2;\ v_i = 0$$

There is no work done by a force applied or a force of friction, therefore, there is no energy added or removed from this system, so mechanical energy is conserved. I do understand there is a force of static friction acting on the cylinder which causes it to rotate, however, because the cylinder does not slide, the force of friction does not cause any energy to be converted to heat or sound.

$$ME_i = ME_f$$



Set the initial point where the cylinder starts at rest and the final point after the cylinder has moved a distance $\Delta d_{\parallel}$. Set the zero line at the center of mass of the cylinder at the final position.

There is no spring so no elastic potential energy initial or final.
Initially the cylinder is at rest so no kinetic energy.
The only type of initial mechanical energy is gravitational potential energy because the cylinder is above the zero line.
There is not final gravitational potential energy because the cylinder is at the zero line.
The cylinder has translational kinetic energy final and rotational kinetic energy final because it is both rotating and its center of mass is translating from one point to another.

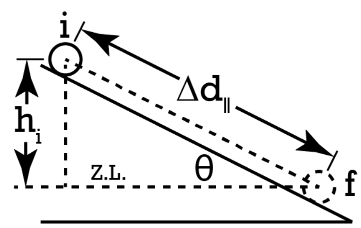
$$\Rightarrow mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_f^2$$

Substitute in the equation for rotational inertia of the solid cylinder.

The cylinder is rolling without slipping so $v_{cm} = R\omega = v_f \Rightarrow v_f^2 = R^2\omega_f^2$

$$\Rightarrow mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{4}mv_f^2 \Rightarrow gh_i = \frac{1}{2}v_f^2 + \frac{1}{4}v_f^2 = \left(\frac{2}{4} + \frac{1}{4}\right)v_f^2 = \frac{3}{4}v_f^2 \Rightarrow v_f^2 = \frac{4}{3}gh_i$$

Everybody brought mass to the party!



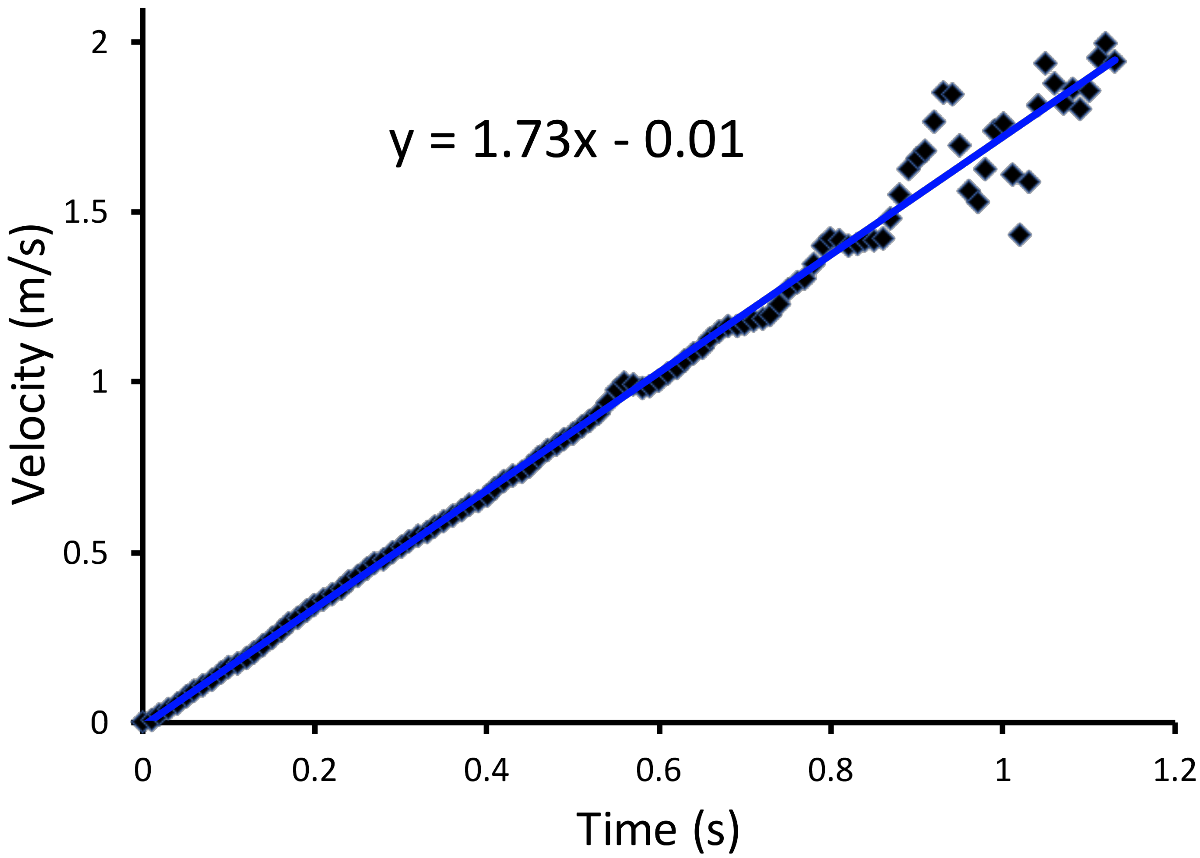
$$\sin\theta = \frac{O}{H} = \frac{h_i}{\Delta d_{\parallel}} \Rightarrow h_i = \Delta d_{\parallel}\sin\theta \Rightarrow v_f^2 = \frac{4}{3}g\Delta d_{\parallel}\sin\theta$$

Acceleration down the incline will be constant, so we can use the uniformly accelerated motion equation:

$$v_{f_{\parallel}}^2 = v_{i_{\parallel}}^2 + 2a_{\parallel}\Delta d_{\parallel} \Rightarrow \frac{4}{3}g\Delta d_{\parallel}\sin\theta = 2a_{\parallel}\Delta d_{\parallel} \Rightarrow a_{\parallel} = \left(\frac{1}{2}\right)\left(\frac{4}{3}g\sin\theta\right) = \boxed{\frac{2}{3}g\sin\theta}$$

Notice this means the only variables which affect the acceleration of a uniform object rolling without slipping down an incline are the planet (acceleration due to gravity), the incline angle θ, and the shape of the object. What I mean by the "shape of the object" is the factor in front of the MR^2 in the rotational inertia equation. Not the mass or radius of the object, but rather just that factor in front of the MR^2.

Demonstration: $g = 9.81\frac{m}{s^2}; \theta = 15.3^\circ \Rightarrow a_{\parallel} = \frac{2}{3}(9.81)\sin(15.3) = 1.72573 \approx 1.73\frac{m}{s^2}$



The slope of velocity a function of time is acceleration, so our experimental acceleration down the incline is also 1.73 m/s^2!!