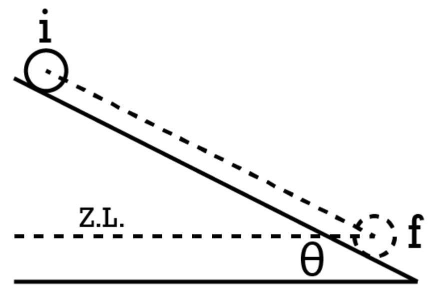
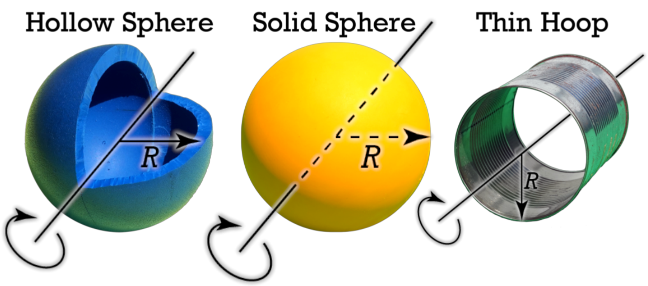
Flipping Physics Lecture Notes:

Which Will Be First? (Rolling Down an Incline)

Example: A hollow sphere, solid sphere, and thin hoop are simultaneously released from rest at the top of an incline. Which will reach the bottom first? Assume all objects are of uniform density.





According to conservation of mechanical energy, each object starts with all gravitational potential energy and ends with translational and rotational kinetic energies.

$$ME_i = ME_f \Rightarrow U_{g_i} = KE_{T_f} + KE_{R_f}$$

In our previous lesson we showed that only the acceleration due to gravity, incline angle, and fraction in front of the MR^2 equation for rotational inertia of the object affect the acceleration down the incline. All three objects are on the same incline and planet, therefore the only difference for each object is the fraction in front of the MR^2 equation for rotational inertia.

Considering the equation for rotational kinetic energy is $KE_R = \frac{1}{2}I\omega^2$, a smaller rotational inertia, I, will mean a smaller rotational kinetic energy. A smaller rotational kinetic energy will mean more energy left over for translational kinetic energy. In other words:

$$I\downarrow \Rightarrow KE_R\downarrow \Rightarrow KE_T\uparrow \Rightarrow v_f\uparrow \Rightarrow \Delta t\downarrow$$

So the smallest fraction for the rotational inertia equation for the object will mean the largest final velocity, which means it will get there first.

Rather than looking up the rotational inertia equations for solid sphere, hollow sphere, and thin hoop, realize you should be able to compare their relative MR^2 fractions empirically using the equation for rotational inertia:

$$I_{\substack{\text{system of}\\ \text{particles}}} = \sum_i m_i r_i^2$$

In other words, the more mass an object has located farther from its axis of rotation, the higher the rotational inertia. Therefore, the order of fraction, X, in the rotational inertia equation should be:

$$I = X\left(MR^2\right)\ \&\ X_{\text{solid sphere}} < X_{\text{hollow sphere}} < X_{\text{thin hoop}}$$

Therefore, the order of final velocities should be:

$$v_{f_{\text{solid sphere}}} > v_{f_{\text{hollow sphere}}} > v_{f_{\text{thin hoop}}} \Rightarrow \Delta t_{\text{solid sphere}} < \Delta t_{\text{hollow sphere}} < \Delta t_{\text{thin hoop}}$$

Therefore, the order of objects reaching the bottom of the incline should be:

- 1st: Solid Sphere
- 2nd: Hollow Sphere
- 3rd: Thin Hoop

Which our demonstration clearly shows to be true. ☺