Flipping Physics Lecture Notes:

Wheel Conservation of Angular Momentum Demonstration and Solution

Let's start with me sitting on a stool free to rotate around a vertical axis through the center of the stool. About that axis, there is no net external torque acting on the stool and me system, so angular momentum of the system is conserved. Initial momentum of the system is zero, so as much as I wave my arms and legs around, I cannot cause the stool to rotate because the angular momentum of the system will stay zero.

However, if I am holding a spinning wheel, the wheel has angular momentum and if I apply a torque to the spinning wheel, the spinning wheel will apply an equal but opposite torque back on me which will cause me to rotate. Note, the angular momentum of the system is still conserved because the two torques, which are equal and opposite, will cancel out, and result in a net torque on the system of zero. In other words, both of these torques, the torque I apply to the wheel and the equal but opposite torque the wheel applies on me, are internal to the person, chair, and wheel system.

Example: A person is holding a spinning wheel while sitting on a stool which is free to rotate. If the person rotates the wheel 180° about a horizontal axis, in terms of the initial angular momentum of the wheel, what is the final angular momentum of the person and stool? (assume no friction)

$$\sum \vec{L}_i = \sum \vec{L}_f;\ \vec{L} = I\vec{\omega}\ \&\ \vec{\omega}_{i\ stool+person} = 0 \Rightarrow \vec{L}_{i\ stool+person} = 0;$$

$$\vec{L}_{f\ stool+person} = X\vec{L}_{i\ wheel};\ X = ?;\ \vec{L}_{f\ wheel} = -\vec{L}_{i\ wheel}$$

$$\sum \vec{L}_i = \sum \vec{L}_f \Rightarrow \vec{L}_{i\ wheel} + \vec{L}_{i\ stool+person} = \vec{L}_{f\ wheel} + \vec{L}_{f\ stool+person}$$

$$\Rightarrow \vec{L}_{i\ wheel} + 0 = -\vec{L}_{i\ wheel} + \vec{L}_{f\ stool+person} \Rightarrow \boxed{\vec{L}_{f\ stool+person} = 2\vec{L}_{i\ wheel}}$$

There is an interesting result to our answer. Notice the direction of the final angular momentum of the stool and person is in the same direction as the initial angular momentum of the wheel. This is true regardless of which direction the person rotates the spinning wheel. ☺

Just so you know, this same physics is how NASA rotates the Hubble Space Telescope. In space there is nothing to push off of to cause a net external torque on the telescope. Therefore, there are near frictionless gyroscopes constantly spinning on the telescope which can be rotated, which in turn rotates the direction the Hubble Space Telescope is pointing. The International Space Station also has gyroscopes to control its rotation. Which is pretty darn cool, if you ask me.