Flipping Physics Lecture Notes:

Acceleration of a Wheel descending on a Rope (Torque Solution)

Example: A rope is wrapped around a bicycle wheel with a rotational inertia of $0.68MR^2$. The wheel is released from rest and allowed to descend without slipping as the rope unwinds from the wheel. In terms of g, determine the acceleration of the wheel as it descends.

$$\sum F_y = F_T - F_g = ma_y \Rightarrow F_T = F_g + Ma_y = Mg + Ma_y \Rightarrow F_T = M\left(g + a_y\right)$$

$$\sum \tau_{\substack{\text{AoR at Center}\\ \text{of Wheel}}} = -\tau_T = I\alpha \Rightarrow -rF\sin\theta = -RF_T\sin 90 = \left(0.68MR^2\right)\left(\frac{a_y}{R}\right)$$

Note: $a_t = r\alpha \Rightarrow a_{cm} = R\alpha = a_y \Rightarrow \alpha = \dfrac{a_y}{R}$

$$\Rightarrow F_T = -0.68Ma_y = M\left(g + a_y\right) \Rightarrow -0.68a_y = g + a_y \Rightarrow -1.68a_y = g \Rightarrow a_y = -\frac{g}{1.68}$$

$$\Rightarrow a_y = -0.59524\text{g} \approx \boxed{-0.60\text{g}}$$

Testing our answer: $a_y = -0.59524g = -\left(0.59524\right)\left(9.81\right) = -5.8393 \approx -5.8\dfrac{m}{s^2}$

Predicted acceleration

$$\Delta y = -0.75m;\ v_{iy} = 0;\ \Delta t = 0.49\text{sec}$$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2 = \frac{1}{2}a\Delta t^2 \Rightarrow a = \frac{2\Delta y}{\Delta t^2} = \frac{\left(2\right)\left(-0.75\right)}{0.49^2} = -6.2474 \approx -6.2\frac{m}{s^2}$$

Measured acceleration