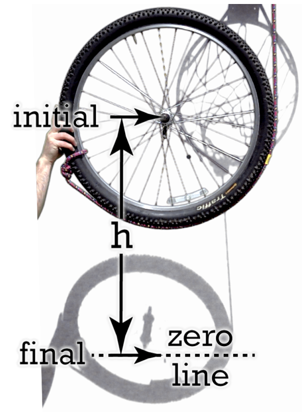
Flipping Physics Lecture Notes:

Acceleration of a Wheel descending on a Rope (Energy Solution)



Example: As shown, a rope is wrapped around a bicycle wheel with a rotational inertia of $0.68MR^2$. The wheel is released from rest and allowed descend without slipping as the rope unwinds from the wheel. In terms of g, determine the acceleration of the wheel as it unwinds from the wheel.[1]

Conservation of Mechanical Energy with initial point where the wheel starts, final point after the wheel has gone down a distance h, and horizontal zero line at the final point.

$$ME_i = ME_f \Rightarrow mgh_i = \frac{1}{2}mv_f^{\,2} + \frac{1}{2}I\omega_f^{\,2}$$

There are no springs in the problem, so no elastic potential energy initial or final.
Initially the wheel is at rest, so it has no initial kinetic energy.
The wheel is initially above the zero line, so it has initial gravitational potential energy.
At the final point the wheel is at the zero line, so it has no final gravitational potential energy.
At the final point the wheel is both rotating and moving translationally, so it has both final translational kinetic energy and final rotational kinetic energy.

$$\Rightarrow Mgh = \frac{1}{2}Mv_f^{\,2} + \frac{1}{2}\left(0.68MR^2\right)\omega_f^{\,2}$$

The mass of the wheel is M, initial height of the wheel is h, and the rotational inertia of the wheel is $0.68MR^2$. Everybody brought mass to the party so we can be equitable and take mass from everybody.

$$v_t = r\omega \Rightarrow v_{cm} = R\omega_f = v_f \Rightarrow v_f^{\,2} = R^2\omega_f^{\,2}$$

Remember the velocity of the center of mass of an object which is rolling without slipping is similar to the tangential velocity equation. The bicycle wheel is rolling without slipping down the rope, so the velocity of the center of mass of the wheel is the final velocity of the wheel.

$$\Rightarrow gh = \frac{1}{2}v_f^{\,2} + 0.34R^2\omega_f^{\,2} = \frac{1}{2}v_f^{\,2} + 0.34v_f^{\,2} = 0.84v_f^{\,2} \Rightarrow v_f^{\,2} = \frac{gh}{0.84}$$

We can substitute $v_f^{\,2}$ in for $R^2\omega_f^{\,2}$ in our equation and solve for $v_f^{\,2}$.

The wheel has a uniform acceleration, so we can use one of the uniformly accelerated motion equations:

$$v_f^{\,2} = v_i^{\,2} + 2a_y\Delta y \Rightarrow \frac{gh}{0.84} = 2a_y\left(-h\right) \Rightarrow \frac{g}{0.84} = -2a_y$$

The initial velocity of the wheel is zero, substitute $\frac{gh}{0.84}$ in for $v_f^{\,2}$, and substitute $-h$ in for Δy because the wheel goes down a distance "h" from the initial to the final points. Everybody brought "h" to the party… And now we can solve for the acceleration of the wheel in the y-direction:

$$a_y = -\frac{1}{2}\left(\frac{g}{0.84}\right) = -0.59524g \approx \boxed{-0.60g}$$

(The negative means the acceleration is down.)

Our answer matches when we did this using torque instead of energy! ☺

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[1] Previously we did this problem using net force and net torque instead of conservation of mechanical energy. https://www.flippingphysics.com/wheel-rope-acceleration-torque.html