Flipping Physics Lecture Notes:

What are the Equations for Kinetic Energy and Angular Momentum of a Point Particle Moving in a Circle?

The equation for the Kinetic Energy of a translationally moving object: $KE_T = \frac{1}{2}mv^2$

The equation for the Kinetic Energy of a rotating object is: $KE_R = \frac{1}{2}I\omega^2$

A point particle which is moving in a circle is moving translationally because its center of mass is not in one constant location and it is rotating because it is moving around an axis of rotation. So which equation for Kinetic Energy should we use?

The equation for the rotational inertia of a system of particles is: $I_{\substack{system\ of \\ particles}} = \sum_i m_i r_i^2$

Therefore, the equation for the rotational inertia of a single particle is: $I_{point\ particle} = mR^2$
Where R is defined as the radius of the circle the point particle is describing.

This means the Rotational Kinetic Energy is: $KE_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mR^2\right)\omega^2$

And the linear velocity of the point particle is a tangential velocity: $v_t = r\omega \Rightarrow v^2 = R^2\omega^2$

Therefore: $KE_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mR^2\right)\omega^2 = \frac{1}{2}mv^2 = KE_T$

The two equations are the equal.
You can use either equation to describe the kinetic energy of a point particle moving in a *circle*.

You can do the same thing with Angular Momentum of a Point Particle versus a Rigid Object with Shape:

$L_{point\ particle} = rmv\sin\theta$ & $L_{\substack{Rigid\ Object \\ with\ Shape}} = I\omega$

$$L_{point\ particle} = rmv\sin\theta = Rm\left(R\omega\right)\sin\left(90\right) = \left(mR^2\right)\omega = I\omega = L_{\substack{Rigid\ Object \\ with\ Shape}}$$

The two equations are the equal.
You can use either equation to describe the angular momentum of a point particle moving in a *circle*.

What if the point particle is moving in an *ellipse*? (Like a satellite)

- Nothing changes in the derivation for kinetic energy, which is a scalar. Therefore, you can still use either the translational or rotational equation for kinetic energy.
- Something does change for angular momentum, which is a vector. The angle between the r vector and velocity is no longer 90 degrees and our derivation is no longer valid.
    - We **cannot** use the equation $L_{\substack{Rigid\ Object \\ with\ Shape}} = I\omega$ for a point particle moving in an ellipse, we have to use $L_{point\ particle} = rmv\sin\theta$.