Flipping Physics Lecture Notes:

Orbital Momentum Conservation

Example: A satellite is in an elliptical orbit. Is its ______ momentum conserved? (a) linear, (b) angular.

(a) Linear momentum is conserved when the net force acting on the system equals zero:

$$\sum \vec{F}_{\substack{exrternal \\ on\ system}} = \frac{\Delta \vec{p}}{\Delta t} = 0 \Rightarrow \sum \vec{p}_i = \sum \vec{p}_f$$

The system in this case is just the satellite. So, is the net force acting on an object in an elliptical orbit equal to zero? Let's draw a free body diagram of the forces acting on the satellite.

FBD: Fg toward center of the primary object which is the object the satellite is orbiting around.

There is only one force acting on the satellite, the force of gravitational attraction caused by the primary, therefore the net force acting on the satellite cannot be zero. So, linear momentum *is not conserved* for an object in an elliptical orbit.

$$\sum \vec{F}_{\substack{exrternal \\ on\ Satellite}} = \frac{\Delta \vec{p}}{\Delta t} \neq 0 \Rightarrow \sum \vec{p}_i \neq \sum \vec{p}_f$$

(b) Angular momentum is conserved when the net torque acting on the system equals zero:

$$\sum \vec{\tau}_{\substack{external \\ on\ system}} = \frac{\Delta \vec{L}_{system}}{\Delta t} = 0 \Rightarrow \sum \vec{L}_i = \sum \vec{L}_f$$

Again, the system is just the satellite. So, is the net torque acting on an object in an elliptical orbit equal to zero? We have already drawn the free body diagram, however, before we can answer the question it is important to remember we have to identify an axis of rotation when working with net torque and angular momentum. Let's define the axis of rotation for the satellite as the center of the primary. Because the only force acting on the satellite is the force of gravitational attraction which points directly toward the center of mass of the primary, the torque caused by this force equals zero, and therefore angular momentum of a satellite in elliptical orbit *is conserved* about the center of mass of the primary.

$$\sum \vec{\tau}_{\substack{external \\ on\ Satellite \\ with\ AoR\ at\ Primary}} = \frac{\Delta \vec{L}_{system}}{\Delta t} = 0 \Rightarrow \sum \vec{L}_i = \sum \vec{L}_f$$

I will point out that, if we assume the orbit is circular, the answers to both of our questions are the same. I simply identified the orbit as elliptical because most orbits are nearly elliptical and very few orbits are nearly circular.