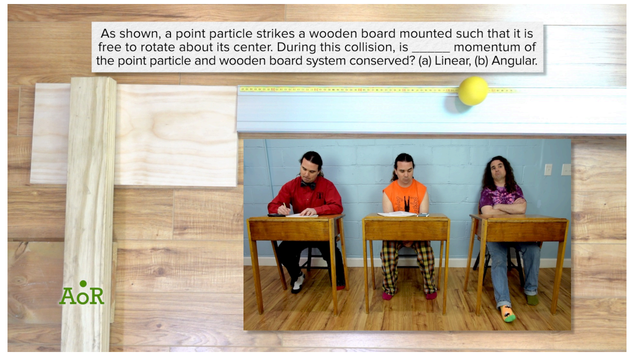
Flipping Physics Lecture Notes:

Are Linear and Angular Momentum Conserved during this Collision?

Example: As shown, a point particle strikes a wooden board mounted such that it is free to rotate about its center. During this collision, is _____ momentum of the point particle and wooden board system conserved? (a) Linear, (b) Angular.



Pictures do not do the example justice.
Please watch the video.
https://www.flippingphysics.com/collision-angular-momentum-conservation.html

Let's start with linear momentum. As the point particle moves toward the wooden board, it is moving to the left and therefore has linear momentum which is to the left. After the collision, the point particle is moving to the right and therefore has linear momentum which is to the right. So, clearly the linear momentum of the point particle changes and is not conserved. However, we are talking about the linear momentum of the point particle and wooden board system. So, what about the linear momentum of the wooden board?

Before the collision the wooden board is not moving, so it has zero linear momentum. After the collision the wooden board is rotating, however, its center of mass is not moving, so it still has zero linear momentum. Therefore, the wooden board has zero change in its linear momentum.

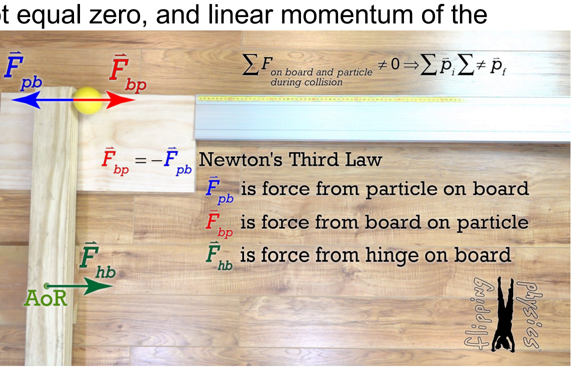
Summing the linear momenta of the wooden board and the point particle, there must have been a change in linear momentum of the system because the linear momentum of the point particle changed, and the linear momentum of the wooden board did not. Therefore, linear momentum of the point particle and wooden board system is **not** conserved.

Looking at the equations, linear momentum is conserved when the net force on the system equals zero:

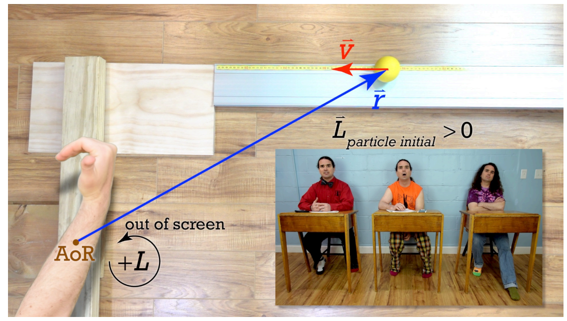
$$\sum \vec{F}_{system} = \frac{\Delta \vec{p}}{\Delta t} = 0 \Rightarrow \Delta \vec{p} = 0 = \vec{p}_f - \vec{p}_i \Rightarrow \vec{p}_{i\ system} = \vec{p}_{f\ system} \text{ or } \sum \vec{p}_i = \sum \vec{p}_f$$

Therefore, the net force on the system in this example must *not* equal zero. When the point particle collides with the board, the point particle applies a force on the board and, according to Newton's Third Law, the board applies an equal but opposite force on the point particle. Those two forces are internal to the system and cancel one another out. So what is another force which is causing the net force on the system to *not* equal zero?

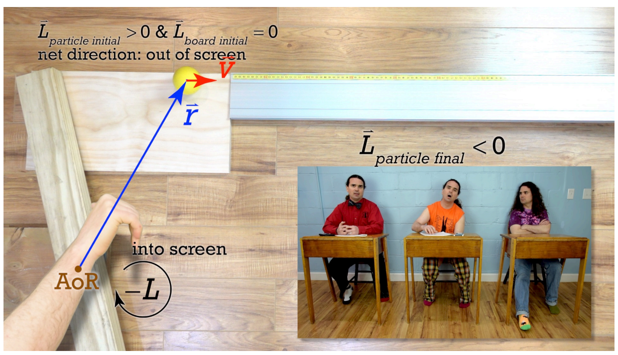
Because the board is attached to a Lazy Susan which allows the board to rotate but not change locations, while the point particle strikes the board, the pivot point applies an external force on the board which prevents the board from entering into translational motion. That means the net external force acting on the board and point particle system during the collision does not equal zero, and linear momentum of the system is not conserved. Please realize this force from the pivot point only acts on the board and point particle system while the point particle is in contact with the board which is only a very short time interval. Also, I will point out, however, that, if the board were not held in place at the pivot point and were instead on a frictionless surface, there would be no net external force acting on the board and point particle system, and linear momentum of the system would be conserved. After that collision, the board would enter in to both rotational and translational motion.

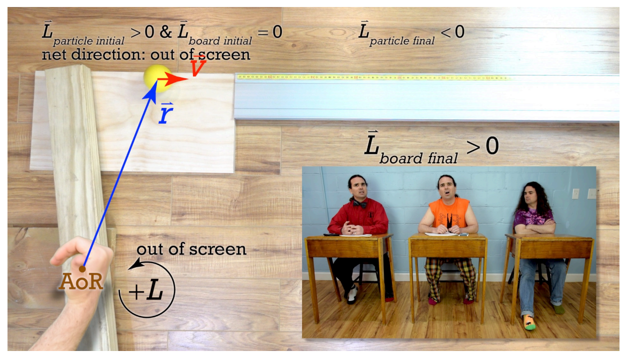


What about angular momentum? Is the angular momentum of the point particle and wooden board system conserved? Going back to looking at the directions of the angular momenta. As the point particle is moving toward the board, it has angular momentum which is, according to the right-hand rule, out of the screen. At this point the board is not moving and has zero angular momentum. So, before the collision, the system has angular momentum which is out of the screen.



After the collision the point particle is moving in the opposite direction and has, according to the right-hand rule, angular momentum which is into the screen. The board now has an angular momentum which is out of the screen. Therefore, the angular momentum before the collision is out of the screen and after the collision is the summation of the point particle's into the screen and wooden board's out of screen angular momenta. In other words, this approach is inconclusive. We cannot tell right now if the angular momentum of the point particle and wooden board system is conserved during this collision. We are going to have to rely on the equations.





Angular momentum is conserved when the net torque on the system equals zero:

$$\sum \vec{\tau}_{system} = \frac{\Delta \vec{L}}{\Delta t} = 0 \Rightarrow \Delta \vec{L} = 0 = \vec{L}_f - \vec{L}_i \Rightarrow \vec{L}_{i\ system} = \vec{L}_{f\ system} \text{ or } \sum \vec{L}_i = \sum \vec{L}_f$$

We already have discussed all the forces acting on the system, so let's see how those cause torques on the system. Let's sum the torques about the axis of rotation of the wooden board.

The two torques caused by the Newton's Third Law Force Pair of forces (the particle on the board and the board on the particle) have the same r vector and their two angels are supplementary, which means they have the same value for sine. Therefore, those two torques are equal and opposite and cancel one another out.

What about the centripetal forces internal to the wooden board which cause the board to rotate? By definition, those centripetal forces are directed in toward the axis of rotation of the board. Therefore, the r vector and force vector are opposite in direction and the angle between those two is 180 degrees. The sine of 180 degrees equals zero. So the centripetal forces internal to the wooden board cause zero torque on the board. The net torque on the system then equals zero and the angular momentum of the point particle and wooden board system *is conserved* about the axis of rotation of the wooden board!