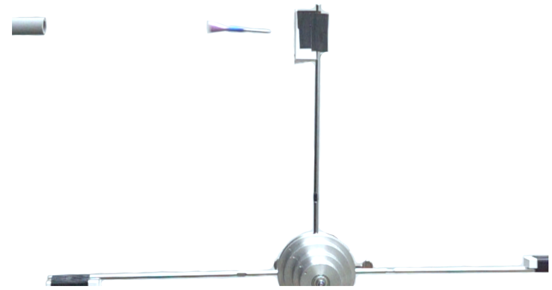
Flipping Physics Lecture Notes:



Point Particle with Rigid Object Collision Conservation of Angular Momentum Demonstration and Problem

Example: As shown, a 5.3 g dart moving horizontally at 16.9 m/s collides with and sticks to a stationary Rotational Inertia Demonstrator a distance of 31.7 cm from the axis of rotation of the RID. What is the final angular velocity of the RID? $I_{RID} = 0.0237\ kg\cdot m^2$

Knowns: $m = 0.0053kg;\ v_{di} = 16.9\frac{m}{s};\ y = 0.317m;\ \omega_{RIDi} = 0;\ I_{RID} = 0.0237kg\cdot m^2;\omega_{RIDf} = ?$

We have already determined that, about the axis of rotation of the rigid object with shape, angular momentum is conserved in a situation where a point particle (the dart) collides with a rigid object with shape (the RID). https://www.flippingphysics.com/collision-angular-momentum-conservation.html

$$\sum \vec{\tau}_{\substack{\text{dart and RID} \\ \text{AoR @ center of RID}}} = \frac{\Delta \vec{L}}{\Delta t} = 0 \Rightarrow \sum \vec{L}_i = \sum \vec{L}_f \Rightarrow L_{di} + L_{RIDi} = L_{df} + L_{RIDf}$$

- We currently have two equations for angular momenta of objects: $L_{\text{point partilce}} = rmv\sin\theta$ & $L_{\substack{\text{Rigid Object} \\ \text{with Shape}}} = I\omega$
- For the dart before the collision, we use the equation for the angular momentum of a point particle: $L_{di} = r_{di}m_d v_{di}\sin\theta_{di}$
    - We are treating the dart as a point particle, so you will see me refer to it as both a "dart" and a "point particle", depending on the situation.
- Before the collision the RID is not moving, so it does not have any angular momentum.
- For the dart after the collision, we have shown that we can use either of the two equations for the angular momentum of the point particle because it is moving in a circle. Let's use: $L_{df} = I_{df}\omega_{df}$ https://www.flippingphysics.com/point-particle-kinetic-energy-and-angular-momentum-equations.html
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- For the RID after the collision, we have to use the equation for the angular momentum of a rigid object with shape: $L_{RIDf} = I_{RID}\omega_{RIDf}$

$$\Rightarrow r_{di}m_d v_{di}\sin\theta_{di} = I_{df}\omega_{df} + I_{RID}\omega_{RIDf}$$

- We have already shown that, while r_{id} and θ_{id} change as the dart moves, y, which equals $r_{id}\sin\theta_{id}$ does not change: $\sin\theta_{di} = \frac{O}{H} = \frac{y}{r_{di}} \Rightarrow y = r_{di}\sin\theta_{di}$ https://www.flippingphysics.com/collision-angular-momentum-conservation.html
- The equation for rotational inertia of a point particle is: $I_{df} = m_d r_{df}^{\ 2}$

$$\Rightarrow y m_d v_{di} = m_d r_{df}^{\ 2}\omega_{df} + I_{RID}\omega_{RIDf}$$

- The dart stays the distance "y" from the axis of rotation after it sticks to the RID: $r_{df} = y$
- Both the dart and the RID are rotating together after the collision: $\omega_{df} = \omega_{RIDf} = \omega_f$

$$\Rightarrow y m_d v_{di} = m_d y^2 \omega_f + I_{RID} \omega_f = \left(m_d y^2 + I_{RID}\right)\omega_f \Rightarrow \omega_f = \frac{y m_d v_{di}}{m_d y^2 + I_{RID}}$$

$$\Rightarrow \omega_f = \frac{(0.317)(0.0053)(16.9)}{(0.0053)(0.317^2) + 0.0237} = 1.1717 \approx \boxed{1.2 \frac{rad}{s}}$$

This is our predicted final angular velocity of the system.

Measured final velocity of the system:

$$Knowns: \Delta\theta = 5.0° \times \frac{2\pi rad}{360°} = \frac{\pi}{36} rad; \Delta t = 69 frames \times \frac{1 \sec}{960 frames} = 0.071875 \sec$$

$$\omega_{measured} = \frac{\Delta\theta}{\Delta t} = \frac{\frac{\pi}{36} rad}{0.071875 \sec} = 1.21414 \frac{rad}{s}$$

$$\text{Percent Difference} = \frac{1.21414 - 1.17171}{\left(1.21414 + 1.17171\right)/2} \times 100 = 3.5566 \approx 3.6\%$$

If you were curious where the rotational inertia of the rotational inertia demonstrator came from, it is the addition of the rotational inertia of it without masses and the rotational inertia of the four 26.1 gram masses which are each 31.0 cm from the axis of rotation. We measured the rotational inertia of the rotational inertia demonstrator in this video: https://www.flippingphysics.com/thin-rod-rotational-inertia.html

$$I_{RID} = I_{RID(no\,masses)} + 4 I_{one\,mass} = I_{RID(no\,masses)} + 4mr^2 = 0.0137 + (4)(0.0260)(0.310)^2 = 0.023733 \approx 0.0237 kg \cdot m^2$$