Flipping Physics Lecture Notes:

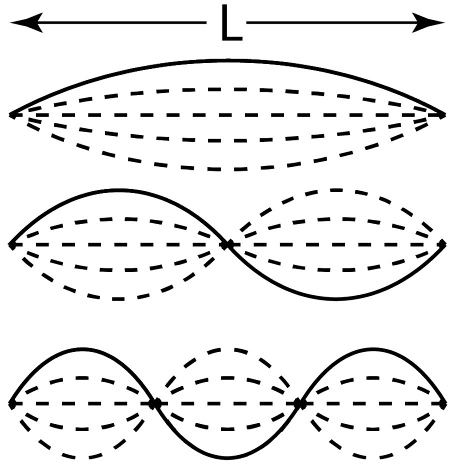
Wind Instrument Frequencies

https://www.flippingphysics.com/wind-instrument.html

Previously we determined the equation for the harmonic frequencies of stringed instruments.
https://www.flippingphysics.com/stringed-instrument.html

$$f_n = n\left(\frac{v}{2L}\right)$$

This is based on the fact that the ends of strings are fixed and therefore must be nodes. This means the standing wave patterns of the first three harmonics look like the diagrams on the right.

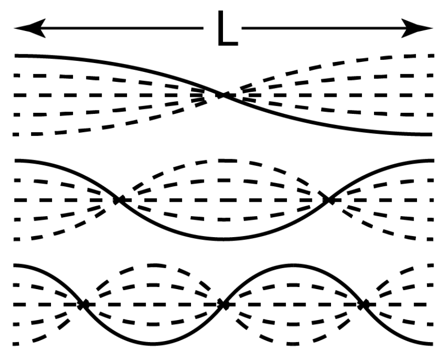


We are now determining the equations for the harmonic frequencies of wind instruments. Wind instruments create a frequency and pitch by creating standing waves in the air columns of instruments. Wind instrument examples are trumpets, tubas, clarinets, flutes, oboes, etc. There are two categories of wind instruments:

- Open Pipe. (Open on both ends.)
- Closed Pipe. (Closed on one end and open on the other.)

An open end creates an antinode and a closed end creates a node.

Let's begin with an open pipe instrument like a flute. Considering both ends are open and an open end is an antinode, this is what the first three standing wave patterns look like:



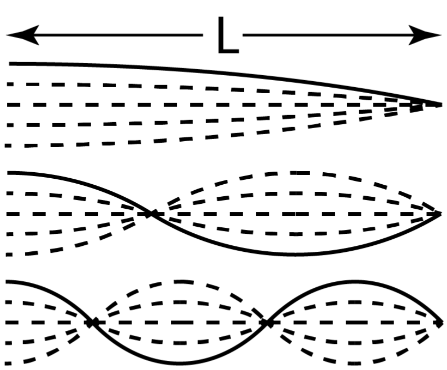
Notice the pattern actually ends up being exactly the same. The first standing wave pattern has half a wavelength in the open pipe. The second standing wave pattern has one wavelength. The third standing wave pattern has one and a half wavelengths. This means the equation for an open pipe instrument is exactly the same as the equation for a stringed instrument.

$$f_n = n\left(\frac{v}{2L}\right)$$

- Same equation.
- Same harmonic number "n".
- v is the speed of sound in air.

This is the equation for the harmonic frequencies of stringed instruments and open pipe wind instruments.

Closed pipe instruments, like a clarinet, have one closed end and one open end. The open end has an antinode and the closed end has a node. This means the first three standing wave patterns look like this:



$$\frac{1}{4}\lambda_1 = L \Rightarrow \lambda_1 = 4L \ \& \ v = f\lambda \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = 1\left(\frac{v}{4L}\right)$$

$$\frac{3}{4}\lambda_3 = L \Rightarrow \lambda_3 = \frac{4L}{3} \ \& \ f_3 = \frac{v}{\lambda_3} = \frac{v}{4L/3} = 3\left(\frac{v}{4L}\right)$$

$$\frac{5}{4}\lambda_5 = L \Rightarrow \lambda_5 = \frac{4L}{5} \ \& \ f_5 = \frac{v}{\lambda_5} = \frac{v}{4L/5} = \frac{5v}{4L} = 5\left(\frac{v}{4L}\right)$$

The equation for the harmonic frequencies for a closed pipe is:

$$f_m = m\left(\frac{v}{4L}\right)$$

- v is the speed of the wave on the string.
- L is the length of the string.
- m is the *harmonic number*.
- m = 1 is the *first harmonic* and the *fundamental frequency*.
- m = 3 is the *third harmonic*.
- m = 5 is the *fifth harmonic* … and so on.
    - Only ***odd*** integer harmonics are possible in a closed pipe instrument.
- Each of the harmonics is an ***odd*** integer multiple of the fundamental frequency.
    - $$f_1 = 1\left(\frac{v}{4L}\right) = \frac{v}{4L} \;\&\; f_3 = 3\left(\frac{v}{4L}\right) = 3f_1 \Rightarrow f_3 = 3f_1 \;\&\; f_5 = 5f_1 \;\&\; etc.$$

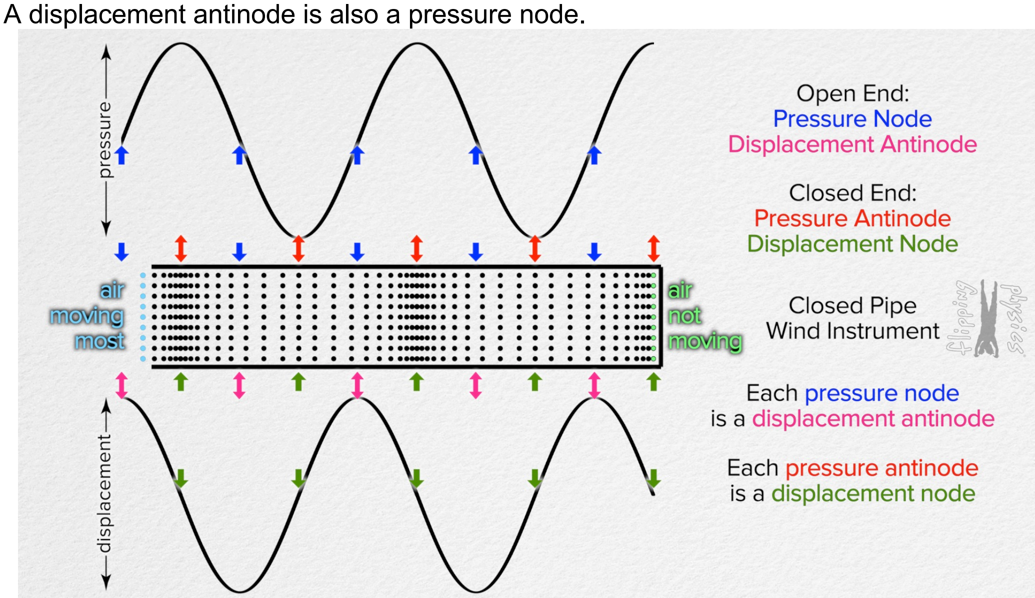
In standing waves in air columns, it is important to understand the difference between:

- Pressure nodes and antinodes.
- Displacement nodes and antinodes.

A closed end is a displacement node because the air cannot move through the closed end. This makes the open end of a pipe a displacement antinode. In other words, all of the illustrations we have used so far are in terms of displacement nodes and antinodes.

An open end is a pressure node because the open end is open to the atmosphere and therefore is constant at atmospheric pressure. This makes the closed end of a pipe a pressure antinode. To be clear:

- A displacement node is also a pressure antinode.
- A displacement antinode is also a pressure node.



Lastly, know that the open end of a pipe is not quite where the location of the pressure node and displacement antinode is. The air does actually oscillate outside the end of the pipe a little bit. In other words, the length of the oscillating column of air in a wind instrument or the effective length of a pipe is a little bit longer than the measured length of the pipe. For a circular cross section pipe, an end correction of 0.6R needs to be added to the length of the pipe for each open end.