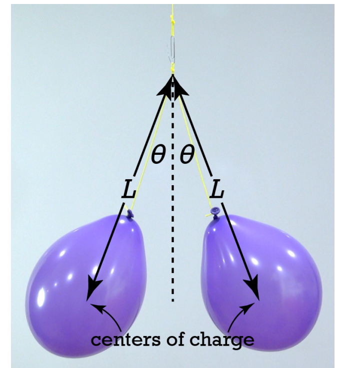
Flipping Physics Lecture Notes:

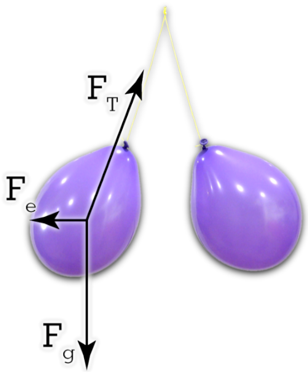
Balloon Excess Charges Experiment

https://www.flippingphysics.com/balloon-charges.html

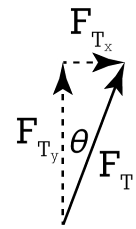


Two 0.0018 kg balloons each have approximately equal magnitude excess charges and hang as shown. If θ = 21° and L = 0.39 m, what is the average number of excess charges on each balloon?

Knowns: $m = 0.0018kg;\ \theta = 21°;\ L = 0.39m;\ q_{avg} = ?$



Draw Free Body Diagram on left balloon.

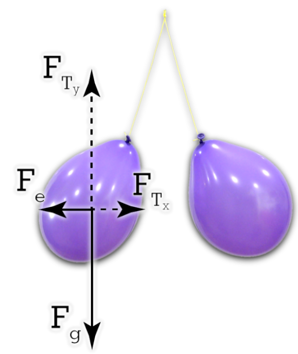


Break Force of Tension into its components.

$$\sin\theta = \frac{O}{H} = \frac{F_{T_x}}{F_T} \Rightarrow F_{T_x} = F_T \sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{F_{T_y}}{F_T} \Rightarrow F_{T_y} = F_T \cos\theta$$

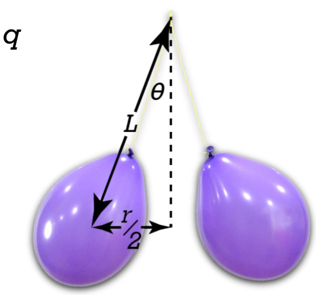
Redraw the Free Body Diagram.



$$\sum F_y = F_{T_y} - F_g = ma_y = m(0) = 0 \Rightarrow F_{T_y} = F_g \Rightarrow F_T \cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_{T_x} - F_e = ma_x = m(0) = 0 \Rightarrow F_{T_x} = F_e \Rightarrow F_T \sin\theta = \frac{kq_1q_2}{r^2}\ \&\ q_1 \approx q_2 \approx q_{avg} \approx q$$

$$\Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{kqq}{r^2} \Rightarrow mg\tan\theta = \frac{kq^2}{r^2} \Rightarrow q = \sqrt{\frac{mgr^2\tan\theta}{k}} = r\sqrt{\frac{mg\tan\theta}{k}}$$



$$\sin\theta = \frac{O}{H} = \frac{r/2}{L} \Rightarrow r = 2L\sin\theta$$

$$\Rightarrow q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}} = 2(0.39)\sin(21)\sqrt{\frac{(0.0018)(9.81)\tan(21)}{8.99\times10^9}} = 2.42719\times10^{-7} \approx 2\times10^{-7}C$$

$$\Rightarrow q_{avg} \approx 2\times10^{-7}C\left(\frac{1\times10^9 nC}{1C}\right) \approx 200nC$$

Is the force of gravity which exists between the two balloons truly negligible?

$$F_e = \frac{kq_1q_2}{r^2} = \frac{kq^2}{4L^2\sin^2\theta} = \frac{\left(8.99\times10^9\right)\left(2.42719\times10^{-7}\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 0.00677827 \approx 7\times10^{-3}N$$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{Gm^2}{4L^2\sin^2\theta} = \frac{\left(6.67\times10^{-11}\right)\left(0.0018\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 2.76582\times10^{-15} \approx 3\times10^{-15}N$$

$$\frac{F_e}{F_g} = \frac{0.00677827}{2.76582\times10^{-15}} = 2.45073\times10^{12} \Rightarrow F_e \approx 2\times10^{12}F_g$$

Given that the electric force is roughly 2 million million times larger than the force of gravity, I would say it is completely reasonable to assume the force of gravity which exists between the two balloons is negligible.

Now we actually answer the question, “what is the average number of excess charges on each balloon?”

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{2.42719\times10^{-7}C}{1.60\times10^{-19}\frac{C}{\text{charge carrier}}} = 1.51699\times10^{12} \approx \boxed{2\times10^{12}\text{ excess charge carriers}}$$