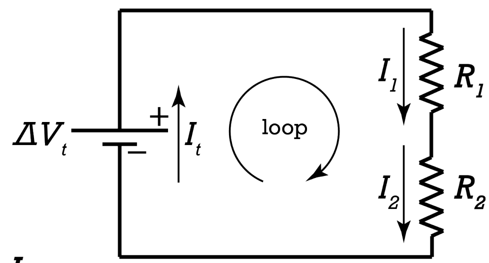
Resistors in Series and Parallel Equation Derivations

https://www.flippingphysics.com/series-parallel-resistors.html



We start with a circuit with a battery and two resisters in series. Because a positive charge would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, the current in this circuit is clockwise or up through the battery and down through each resistor. Let's label those currents as the terminal current through the battery and current 1 and current 2 through their respective resistors. Hopefully you recognize that each charge on the wire has to go through all three of these circuit elements, therefore all of these currents are equal: $I_t = I_1 = I_2$

According to Kirchhoff's Loop Rule, a charge moving all the way around a loop in a circuit must end with the same electric potential energy it started with, therefore, the electric potential difference all the way around a loop is equal to zero. If we define the loop in a clockwise direction in our circuit, Kirchhoff's Loop Rule looks like this:

$$\Delta V_{loop} = 0 = \Delta V_t - \Delta V_1 + \Delta V_2 \Rightarrow \Delta V_t = \Delta V_1 + \Delta V$$

Because electric potential difference equals current times resistance, we can substitute current times resistance for each of the electric potential differences. For the battery, the terminal voltage equals the current at the terminals of the battery times the equivalent resistance of the electrical load. The electrical load in this circuit is the two resistors in series.

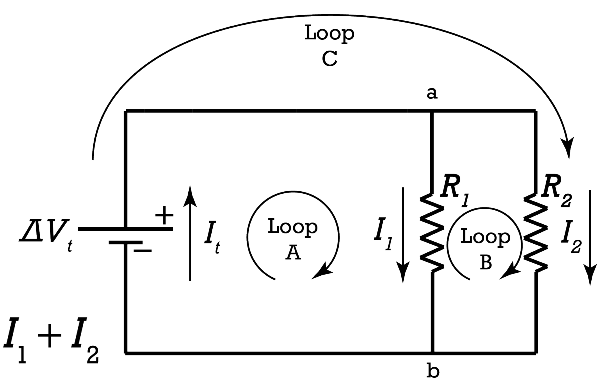
$$\Delta V = IR$$

$$\Rightarrow I_t R_{eq} = I_1 R_1 + I_2 R_2$$

All the currents are the same, so they cancel out and the equivalent resistance for two resistors in series equals the sum of those two resistors. We could perform this experiment with as many resistors in series as we wanted to, and the equivalent resistance would always be the sum of the resistances.

$$\Rightarrow R_{eq} = R_1 + R_2 \Rightarrow \boxed{R_{series} = R_1 + R_2 + R_3 + \ldots}$$

Note: In series circuit elements currents are the same and electric potential differences add.



Now let's do a circuit with a battery and two resistors in parallel. Again, the current directions are up through the battery and down through each of the resistors. There are two junctions in the circuit; junction a and junction b. Using Kirchhoff's Junction Rule, which is a result of conservation of charge, the fact that every charge that goes into the junction must come out of the junction, for junction a we get:

$$\sum I_{in} = \sum I_{out} \Rightarrow I_t = I_1 + I_2$$

We can define three loops for Kirchhoff's Loop Rule as shown in the figure. Remembering that electric potential goes up as you go from the negative to the positive terminals of the battery and, as you go in the direction of current across a resistor, the electric potential goes down; these are the equations for loop A and loop C:

$$\Delta V_{Loop\ A} = 0 = \Delta V_t - \Delta V_1 \Rightarrow \Delta V_t = \Delta V_1$$

$$\Delta V_{Loop\ C} = 0 = \Delta V_t - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_2$$

Notice then that all of the electric potential differences in this circuit are the same. And because electric potential difference equals current times resistance, current equals electric potential difference divided by

$$\Rightarrow \Delta V_t = \Delta V_1 = \Delta V_2 \ \& \ \Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$$

resistance. Therefore, we can combine these equations to solve for the equivalent resistance of the two resistors in parallel:

$$I_t = I_1 + I_2 \Rightarrow \frac{\Delta V_t}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

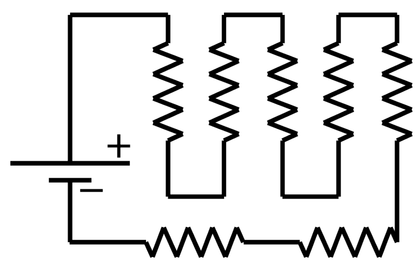
We could perform this experiment with as many resistors in parallel as we want, and the equivalent resistance will always be equal to the inverse of the sum of the inverses of all the resistors in parallel.

$$R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \right)^{-1}$$

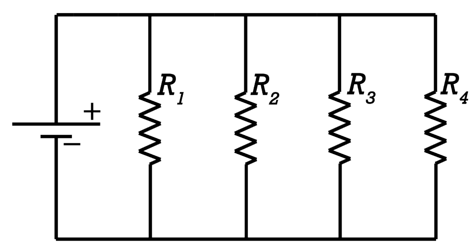
Note: In parallel circuit elements currents add and electric potential differences are the same.

So, notice that adding resistors in series increases the net resistance of the resistors and adding resistors in parallel decreases the net resistance of the resistors. Think of it this way, by adding a resistor in series, you are adding resistance to the path the charges to go through which, no matter how small that resistance is, still increases the resistance. When you add a resistor in parallel, you are adding an additional path for the charges have to go through and therefore, no matter how large the resistor is which you are adding in parallel, the addition of another pathway for the charges to travel decreases the overall resistance.

It may seem pretty obvious in simple circuits like the ones we just went through; however, it is important to identify when circuit elements are in series, parallel, or neither. Let's start with series. If every charge that goes through one element also has to go through the other element, those two circuit elements are in series. For example, all of the resistors in the following circuit are in series. This is because every charge in the circuit has to pass through every one of the resistors in the circuit.



Circuit elements which are in parallel all have the same electric potential difference. For example, all the resistors in the following circuit have the same potential at the top and bottom of the resistor, so their electric potential differences are the same. Another way to look at this is that if the charges are split between resistors and then all the charges come back together again, the resistors are in parallel.



If you look at the next circuit, it appears to be different, however, the top of each resistor (or right side in the case of resistor 4) are all at the same electric potential and the bottom of each resistor (or left side in the case of resistor 4) are at the same electric potential, therefore, all four of these resistors are still in parallel. In fact, this is the same circuit as before, it has simply been drawn slightly differently.